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# Towards a final explanation of the contact angle saturation in electrowetting based on the exploration of the Maxwell stress tensor at a triple point

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**Abstract** – We present a rigorous analysis of the (maximal) electric force at a conductive wedge formed by the wetting angle of a liquid droplet on an insulated electrode using the Maxwell stress tensor, yielding a final explanation for the so-called *contact angle saturation* in electrowetting. When applying a voltage on the liquid droplet placed on a insulated electrode the resulting forces will be confined to the tip of the wedge – represented by a *triple junction* of the three adjacent phases: conductive liquid, dielectric and surrounding air. These confined forces tend to drag the droplet over the hydrophobic dielectric surface (*electrowetting effect*) causing a deformed liquid surface with a decreased contact angle. Forces acting solely on the triple point while being described by corresponding surface tensions together with an interfacial shear force define the realm of the approximate *Young-Lippmann* formalism, where the voltage dependent contact angle becomes the major parameter for the emergent drag force. It is though not surprising that the *contact angle saturation* at a minimal angle (yielding maximal drag force) is subject to a still unresolved debate relating its origin to various *microscopic* but rather disconnected mechanisms. Here we present a *macroscopic* explanation of the *contact angle saturation* that has the potential to predominate the *microscopic* ones due to emergent electrostatic field singularities in the *triple point.*

In our theoretical 2D analysis we calculated the electric field and flux density in a close radial neighborhood around the triple point relying on their well-known fractional order local dependence. Using the Maxwell stress tensor together with renormalization techniques, it is shown that the estimated electrostatic «pressure» (force/m$^2$) in the singularity together with the adjacent much weaker ones are strongly dependent on the contact angle, where the horizontal component of this resulting pressure changes direction (with a very steep zero crossing) at a contact angle of 80°. Below this angle any droplet transport is inhibited and so the decrease of the contact angle, rendering this limiting value to *contact angle saturation*. It's worth mentioning that the zero crossing is of purely geometrical nature, whereas voltage levels, permittivity and thickness of the dielectric layer have no direct influence but rather on the steepness of the zero crossing, providing a sensitive argument to the variations in the reported experimental values.

# Towards a final explanation of the contact angle saturation in electrowetting based on the exploration of the Maxwell stress tensor at a triple point

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## 1. Introduction

We present a rigorous analysis of the (maximal) electric force at a conductive wedge formed by the wetting angle of a liquid droplet on an insulated electrode using the Maxwell stress tensor yielding a final explanation for the so-called *contact angle saturation* in electrowetting [1,2]. When applying a voltage on the liquid droplet placed on a insulated electrode the resulting forces will be confined to the tip of the wedge – represented by a *triple junction* [3,4] of the three adjacent phases: conductive liquid, dielectric and surrounding air. These confined forces tend to drag the droplet over the hydrophobic dielectric surface (*electrowetting effect*) causing a deformed liquid surface with a decreased contact angle. Forces acting solely on the triple point of the liquid droplet while being described by corresponding surface tensions together with an interfacial shear force define the realm of the approximate *Young-Lippmann* formalism [1], where the voltage dependent contact angle becomes the major parameter for the emergent drag force.

In practice, however, the contact angle of the liquid droplet decreases with the increasing voltage until a saturation point (at about 80°) is reached [1,2]. The contact angle saturation at a minimal angle (yielding maximal drag force) is subject to a still unresolved debate relating its origin to various microscopic but rather disconnected mechanisms. Here we present a macroscopic explanation of the contact angle saturation that has the potential to predominate the microscopic ones due to emergent electrostatic field singularities in the triple point.

## 2. Electrowetting

Young equation
for surface tension

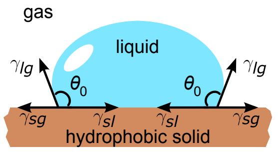
$$\gamma_{sg} - \gamma_{sl} - \gamma_{lg} \cdot \cos\theta_0 = 0$$

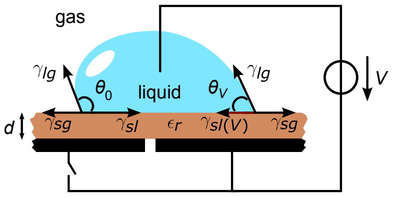
$$\cos\theta_0 = \frac{\gamma_{sg} - \gamma_{sl}}{\gamma_{lg}}$$

Young-Lippmann equation

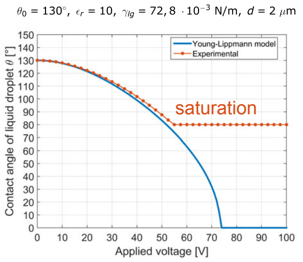
$$\gamma_{sl(V)} = \gamma_{sl(V=0)} - \frac{\epsilon \cdot V^2}{2d}$$

$$\cos\theta_V = \cos\theta_0 + \frac{\epsilon \cdot V^2}{2d \cdot \gamma_{lg}}$$



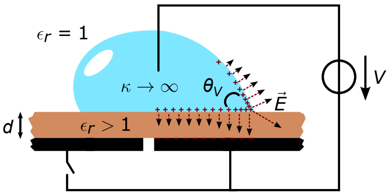


Theoretical example of
contact angle saturation



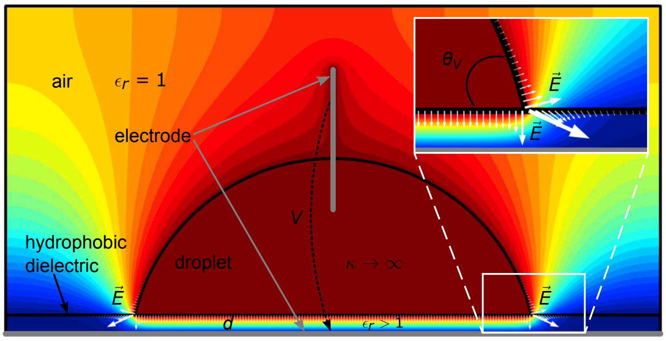
Electromechanical approach

$$\vec{F}_{el} = \oint_{\partial V} \overleftrightarrow{T}_{el} \cdot d\vec{A}$$



## 3. Electrostatic pressure on droplet

Electric field distribution on a surface of the perfectly conducting droplet (FEM)



Calculation of the electrostatic pressure using the Maxwell stress tensor

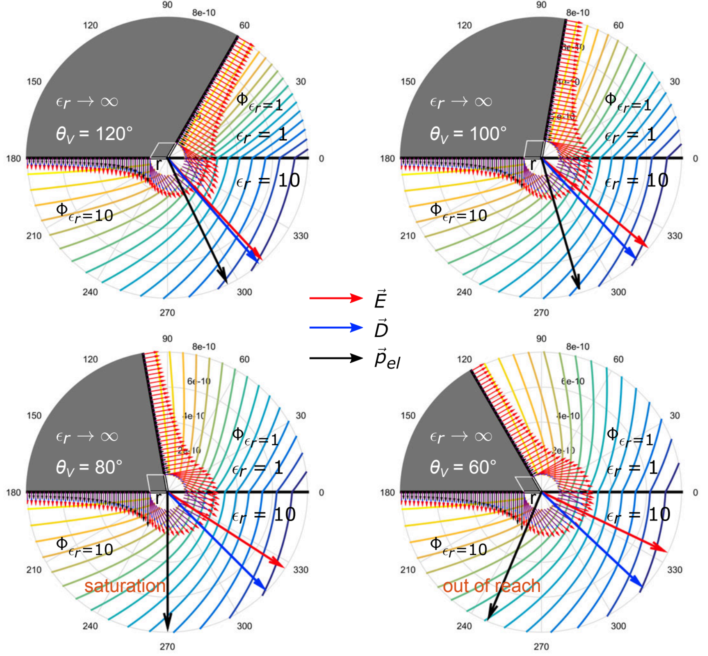
$$\vec{p}_{el} = \frac{d\vec{F}_{el}}{dA} = \overleftrightarrow{T}_{el}(\vec{E},\vec{D}) \cdot \vec{n}$$

Singular fields $\vec{E}$ and $\vec{D}$ in a droplet wedge with angle $\theta_V$ – analytical estimations

$$\Phi \propto r^{\nu} \qquad (\vec{E},\vec{D}) \propto r^{\nu-1} \qquad \vec{p}_{el} \propto r^{2(\nu-1)} \qquad \nu = \nu(\theta,\epsilon_r)$$

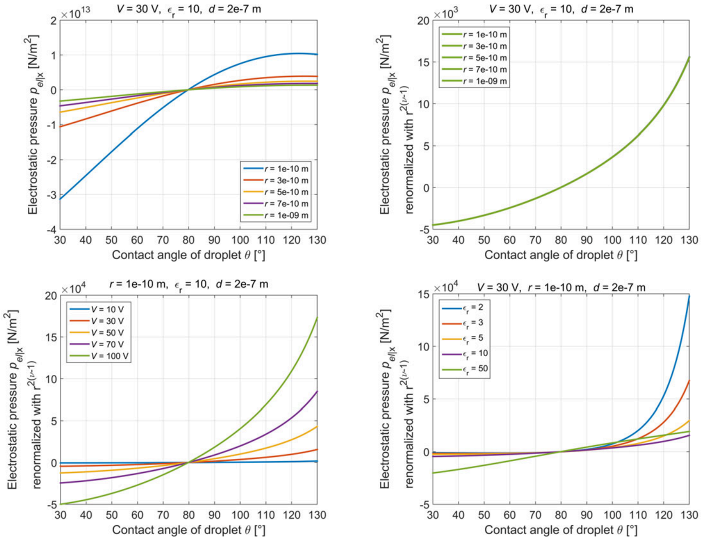
## 3.a. Field analysis

Determination of $\vec{E}$, $\vec{D}$, $\vec{p}_{el}$ in a close radial neighborhood around the droplet triple point with $r$ = 100 pm



## 3.b. Renormalization and results

Horizontal component of the resulting electrostatic pressure $p_{el|x}$ on the droplet wedge in dependency of the triple junction "radius" $r$, voltage $V$ and dielectric permittivity $\epsilon_r$



## 4. Conclusion

- The electrostatic pressure in the singularity together with the adjacent much weaker ones are strongly dependent on the contact angle, where the horizontal component of this resulting pressure changes direction (zero crossing) at a contact angle of 80°.
- The zero crossing is of purely geometrical nature, whereas $V$, $\epsilon_r$ and the dielectric thickness $d$ have an influence on the steepness of the zero crossing, providing a sensitive argument to the variations in the experimental values.
- Numerical results from FEM-based simulations with COMSOL Multiphysics showed the same trend of the zero crossing.

## 5. Literature

[1] Cho et al., "Creating, Transporting, Cutting and Merging Liquid Droplets by Electrowetting-Based Actuation for Digital Microfluidic Circuits", *Journal of Microelectromechanical Systems*, vol. 12, no. 1, pp. 70-80, February 2003.
[2] J. Berthier, *Microdrops and Digital Microfluidics*. Norwich, NY, USA: William Andrew, 2008.
[3] L. Schächter, "Analytic expression for triple-point electron emission from an ideal edge", *Applied Physics Letters – American Institute of Physics*, vol. 72, no. 4, pp. 421-423, January 1998.
[4] T. Takuma and T. Kawamoto, "Field Enhancement at a Triple Junction in Arrangements Consisting of Three Media", *IEEE Transactions on Dielectrics and Electrical Insulation*, vol. 14, no. 3, pp. 566-571, June 2007.

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# Discovery

**Poster**

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These confined forces tend to drag the droplet over the hydrophobic dielectric surface (electrowetting effect) causing a deformed liquid surface with a decreased contact angle. Forces acting solely on the triple point while being described by corresponding surface tensions together with an interfacial shear force define the realm of the approximate Young-Lippmann formalism, where the voltage dependent contact angle becomes the major parameter for the emergent drag force. It is though not surprising that the contact angle saturation at a minimal angle (yielding maximal drag force) is subject to a still unresolved debate relating its origin to various microscopic but rather disconnected mechanisms.

Here, we present a macroscopic explanation of the contact angle saturation that has the potential to predominate the microscopic ones due to emergent electrostatic field singularities in the triple point. In our theoretical 2D analysis, we calculated the electric field and flux density in a close radial neighbourhood around the triple point relying on their well-known fractional order local dependence. Using the Maxwell stress tensor together with renormalization techniques, it is shown that the estimated electrostatic «pressure» (force/m$^2$) in the singularity together with the adjacent much weaker ones are strongly dependent on the contact angle, where the horizontal component of this resulting pressure changes direction (with a very steep zero crossing) at a contact angle of 80°. Below this angle any droplet transport is inhibited and thus any further decrease of the contact angle.

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