Efficient Analysis Method of Light Scattering by a Grating of Plasmonic Nanorods

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I. Introduction.

II. The Lattice Sums Technique combined with the transition matrix (T-matrix) and recursive algorithm.

III. Scattering by plasmonic grating and its application in refractive index sensing.

IV. Conclusions.

INTRODUCTION

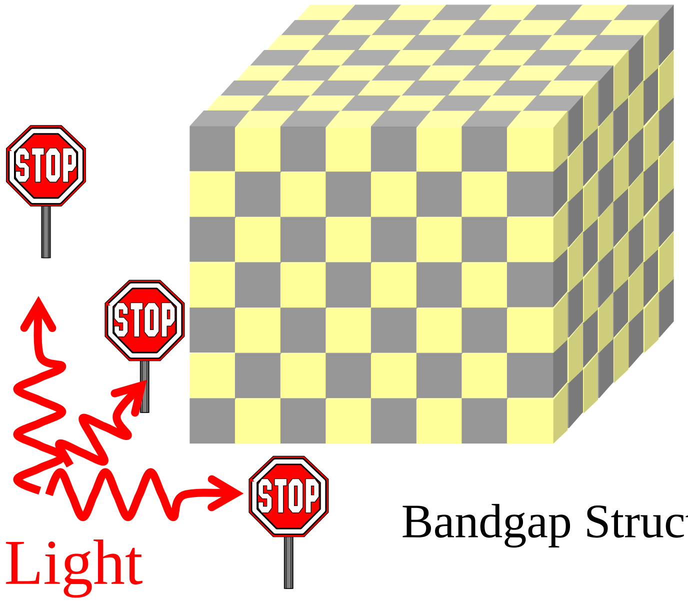
- The enhanced surface plasmon resonance in noble metallic systems at optical frequencies is expected to be a promising issue for realizing excellent scatterers and absorbers of the visible light.

- The surface plasmons characterize the unique response in collective motions of electrons on a metal-dielectric interface, which is allowed when the permittivity of the metal is negative for the wavelength of excitation.

- Surface plasmons are accompanied by localized enhancement of the electromagnetic field. This leads to developing ***metallic nanostructures that can control light at the nanoscale in direct analogy*** to traditional optical components such as lens, mirrors and waveguides. Many of these applications rely on a ***planar geometry*** in practice.

1. P. Berini, "Long-range surface plasmon polaritons," *Adv. Opt. Photonics*, vol. 1, pp. 484-588, 2009.
2. O. Hess, J. Pendry, S. Maier, R. Oulton, J. Hamm and K. Tsakmakidis, "Active nanoplasmonic materials," *Nature Materials*, vol. 11, pp. 573-584, 2012.

Periodic and Bandgap Structures



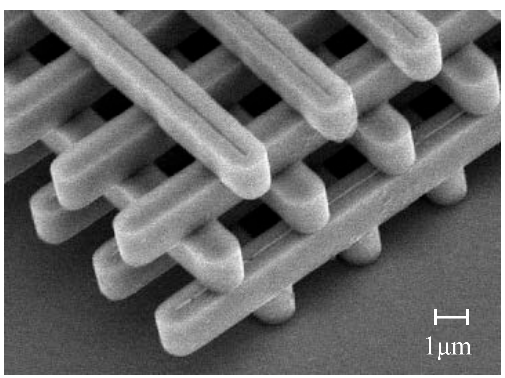
Bandgap Structures

Unique feature to localize electromagnetic waves to specific arrays and to guide along certain directions at restricted frequencies.

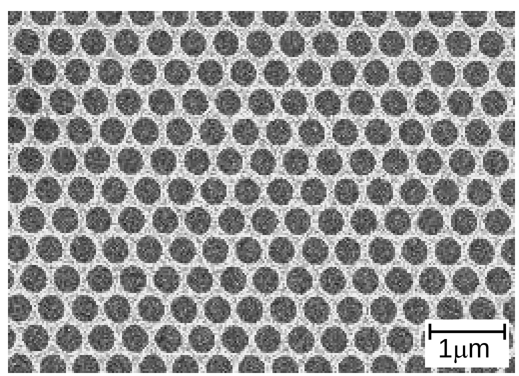
R. A. Silin and V. P. Sazonov, Slow-Wave Structures, Moscow: Soviet Radio, 1966.

E. Yablonovitch, “Inhibited spontaneous emission in solid-state physics and electronics,” Phys. Rev. Lett., vol.58, p.2059, 1987

S. John, “Strong localization of photons in certain disordered dielectric superlattices,” Phys. Rev. Lett., vol.58, p.2486, 1987



3D Woodpile type Crystals (Sandia)



2D Air-hole type Crystals (NEC)

Experimental Examples

Application in:

- Filters
- Waveguides
- Optical Fibers
- Antenna Substrate/Cover
- Amplifiers

- Scattering of TE polarized plane wave by plasmonic grating coupled to isotropic or anisotropic slab is rigorously investigated utilizing the recursive algorithm combined with the Lattice Sums technique.

- All Floquet modes and their interactions through the scattering by plasmonic grating and isotropic/anisotropic media are taken into account. It enables to give a physical insight into the coupling process.

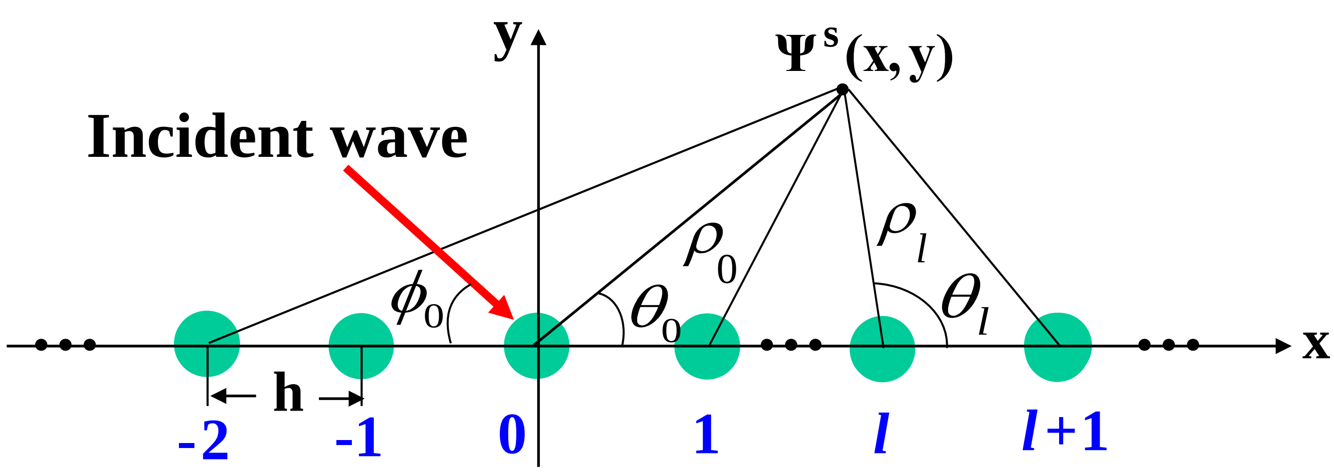
- Using the proposed formulation we can ***in a very short time*** analyze various models. When the desired characteristics are observed, we will be able to implement the geometrical parameters under which the desired characteristics are obtained, in the numerical and experimental studies of the real world structures.

It will substantially decrease time, effort and expenses needed for the detailed numerical and experimental investigations.

LATTICE SUMS TECHNIQUE COMBINED WITH RECURSIVE ALGORITHM

Single layer of periodic array

$$\Psi(x,y)=\begin{cases}E_z & \text{for TM wave}\\ H_z & \text{for TE wave}\end{cases}$$

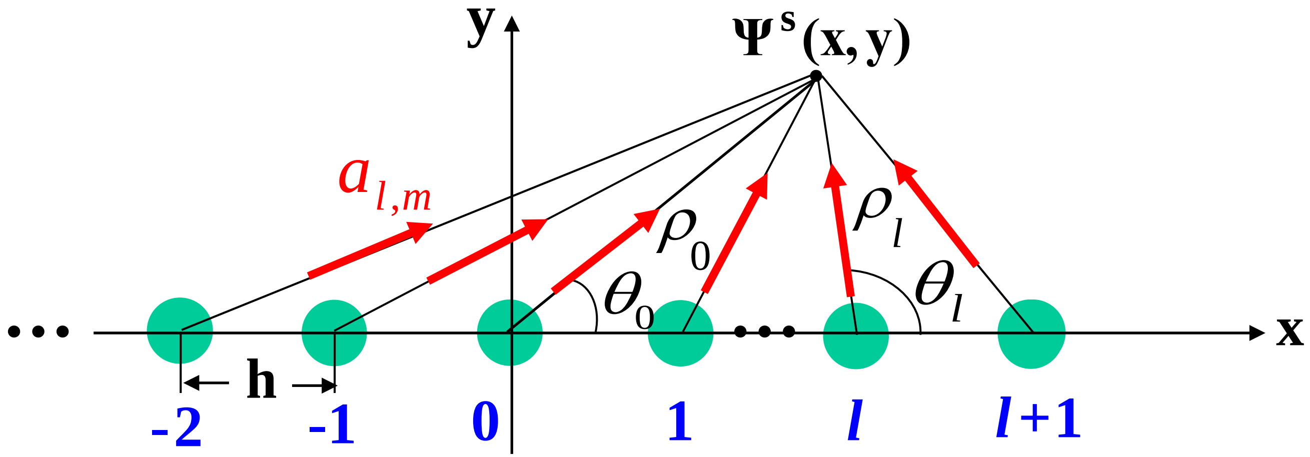


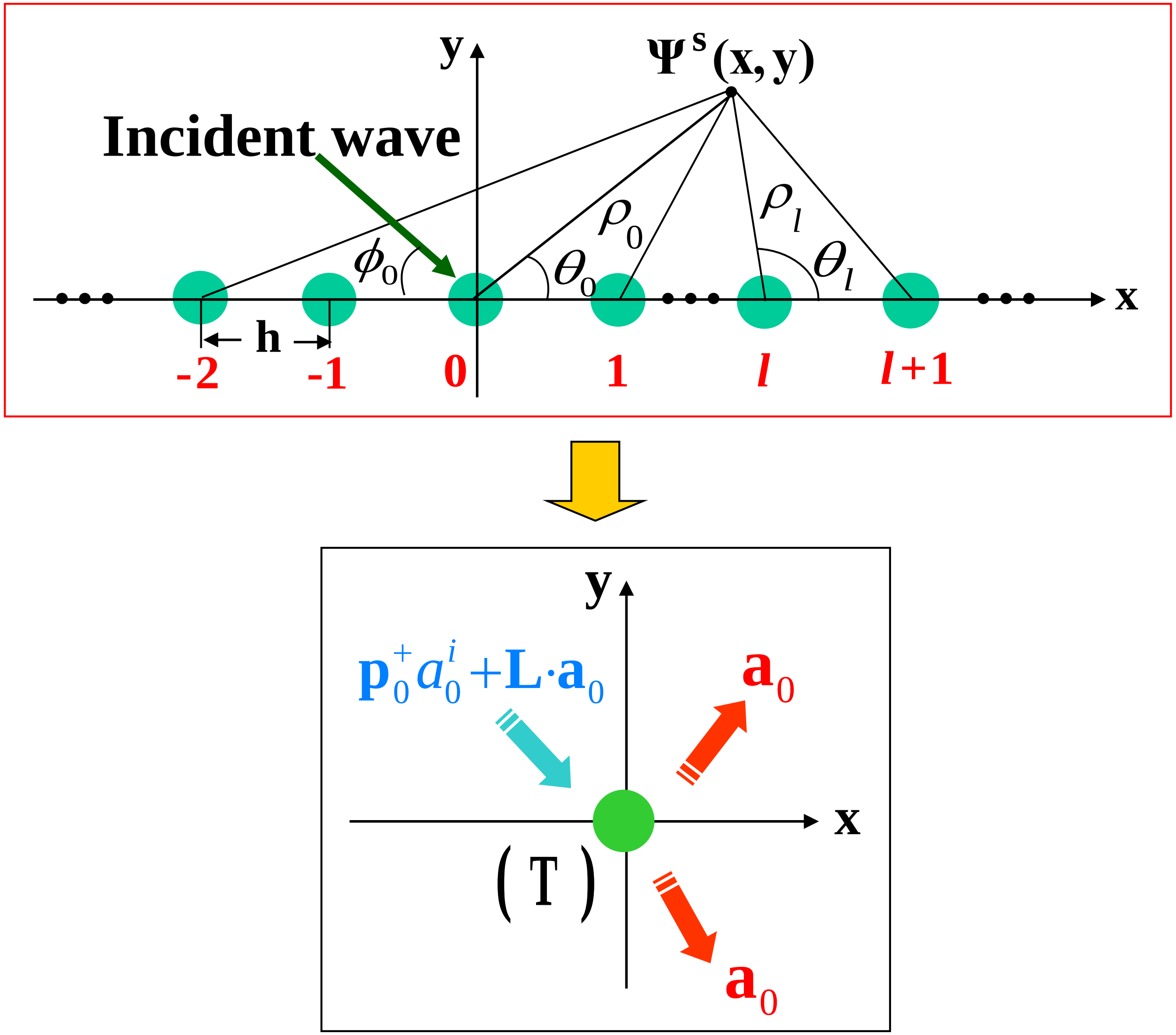
Cross section of a periodic array of cylindrical objects

$$\Psi^s(x,y) = \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_{l,m} \, \mathrm{H}_m^{(1)}(k\rho_l)\, \mathrm{e}^{im\theta_l}$$

$$\rho_l = \sqrt{(x-lh)^2 + y^2}, \quad \cos\theta_l = \frac{x-lh}{\rho_l}$$

$a_{l,m}$ **: Unknown amplitudes of multipole fields scattered from l- th cylinder**





Incident wave
y
$\Psi^s(x,y)$
ρ_0
ρ_l
ϕ_0
θ_0
θ_l
x
h
-2
-1
0
1
l
$l+1$
y
$\mathbf{p}_0^+ a_0^i + \mathbf{L}\cdot\mathbf{a}_0$
$\mathbf{a}_0$
x
(T)
$\mathbf{a}_0$

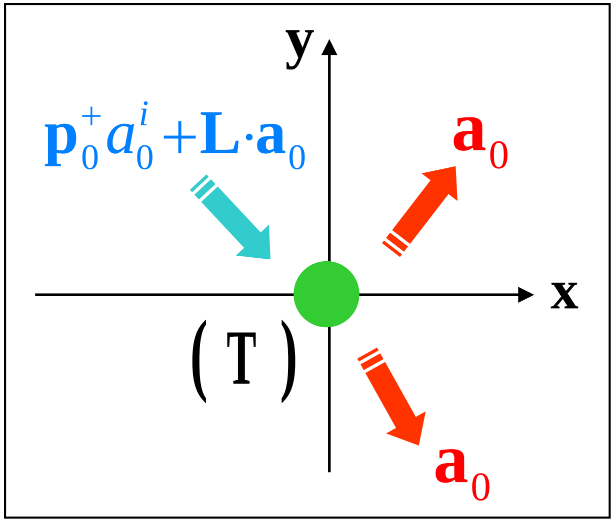
Total field

$$\Psi(x,y) = \Psi^i(x,y) + \Psi^s(x,y)$$
$$= \mathbf{\Phi}_0^{\mathrm{T}} \cdot (\mathbf{p}_0^+ a_0^i + \mathbf{L} \cdot \mathbf{a}_0) + \mathbf{\Psi}_0^{\mathrm{T}} \cdot \mathbf{a}_0$$

⇒ $$\mathbf{a}_0 = \mathbf{T} \cdot (\mathbf{p}_0^+ a_0^i + \mathbf{L} \cdot \mathbf{a}_0)$$

$$\mathbf{L} = [L_{mn}] = [S_{m-n}(k_0 h, k_{x0} h)]$$

T : **T-matrix of a cylindrical object in isolation**



Amplitudes of scattered field

⇒ $$\mathbf{a}_0 = \bar{\bar{\mathbf{T}}} \cdot \mathbf{p}_0^+ a_0^i$$

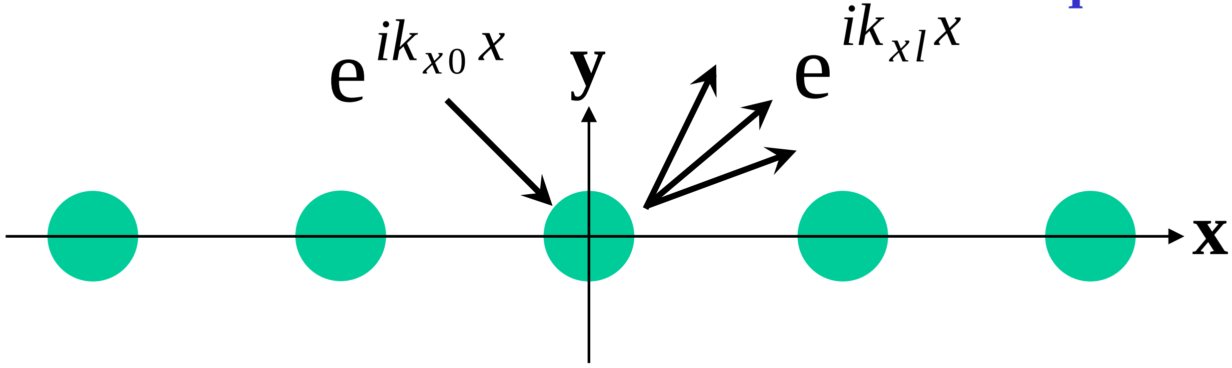
$$\bar{\bar{\mathbf{T}}} = (\mathbf{I} - \mathbf{T} \cdot \mathbf{L})^{-1} \cdot \mathbf{T}$$

$\bar{\bar{\mathbf{T}}}$: **T-matrix of a periodic array of cylindrical objects**

Single array

$$e^{ik_{x0}x} \Rightarrow e^{ik_{xl}x}, \quad k_{xl} = k_{x0} + \frac{2l\pi}{h}$$

Floquet Theorem

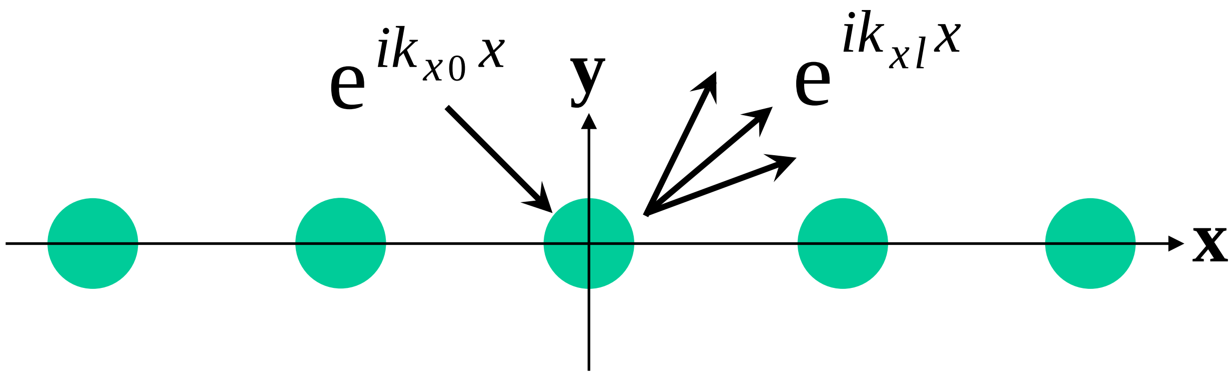


Reflected fields $(y>0)$

$$\psi_l^r(x,y) = r_l^{(+)} a^i e^{i(k_{xl}x + k_{yl}y)}, \quad r_l^{(+)} = \boldsymbol{u}_l^{(+)T} \cdot \overline{\overline{\mathbf{T}}} \cdot \boldsymbol{p}^-$$

$r_l^{(+)}$: 0-th incident wave ➡ l-th reflected wave

Single array



Transmitted fields $(y<0)$

$$\psi_l^t(x,y) = f_l^{(-)} a^i \, e^{i(k_{xl}x - k_{yl}y)}, \qquad f_l^{(-)} = \delta_{l0} + \boldsymbol{u}_l^{(-)T} \cdot \overline{\overline{\mathbf{T}}} \cdot \boldsymbol{p}^-$$

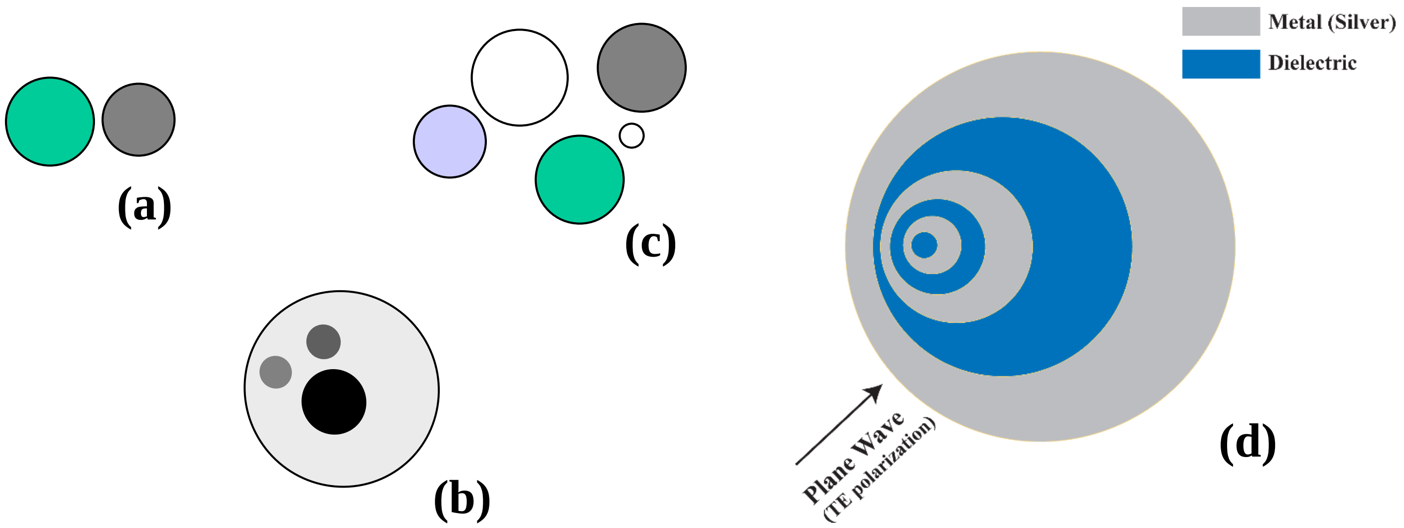
$f_l^{(-)}$: 0-th incident wave ➡ l-th transmitted wave

$$\boldsymbol{u}_l^{(\pm)} = [u_{lm}^{(\pm)}] = \left[\frac{2(-i)^m}{k_0 h \sin\alpha_l} e^{\pm i m \alpha_l} \right]$$

$$\cos\alpha_l = \frac{k_{xl}}{k_0}, \qquad \mathrm{Im}\{\sin\alpha_l\} \geq 0$$

Transition Matrix (T-matrix)

➡ When the cylindrical objects are circular cylinders, the T-matrix per unit cell can be obtained in closed form.



➡ If the cylindrical objects with rectangular, triangular or other arbitrary cross sections are treated, the T-matrix must be calculated by using other numerical methods.

When the electromagnetic scattering by a periodic structure is formulated, one needs inevitably to calculate the infinite series of Hankel functions (called the lattice sums). The LSs are complex coefficients L_n, namely, series involving higher-order Hankel functions of the first type $H_m^{(1)}$:

$$L_n(k_0 h, k_x h) = \sum_{\ell=1}^{\infty} \mathrm{H}_n^{(1)}(\ell k_0 h)[\mathrm{e}^{ik_x h\ell} + (-1)^n \, \mathrm{e}^{-ik_x h\ell}]$$

L_n is independent of the polarization of the incident field and the individual configuration of the scatterers: it uniquely characterizes a periodic arrangement of objects.

$$k_{x0} = \beta$$

Real wavenumber [1]:
Slow converging series

$$k_{x0} = \beta - j\alpha$$

Complex wavenumber [2,3]:
Exponentially diverging series

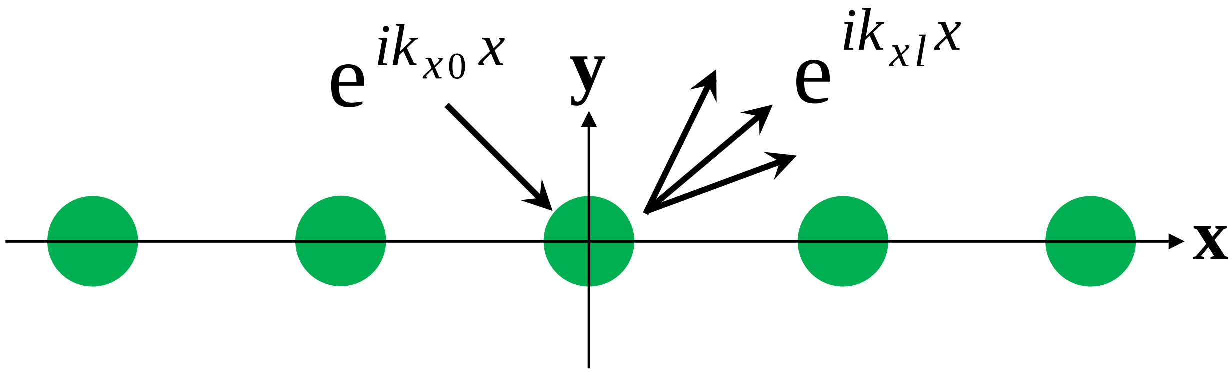
[1] K. Yasumoto and K. Yoshitomo, *IEEE TAP*, vol.47, pp.1050-1055, 1999.

[2] V. Jandieri, P. Baccarelli, C. Ponti and G. Schettini, “Full-wave analysis of leaky modes in 2-D EBG waveguides,” *Proceedings of the 11-th European Conference on Antennas and Propagation*, Paris, France, pp. 3235-3236, March, 2017.

[3] P. Baccarelli, V. Jandieri, G. Valerio and G. Schettini, “Efficient computation of the lattice sums for leaky waves using the Ewald method,” *Proceedings of the 11-th European Conference on Antennas and Propagation*, Paris, France, pp. 3233-3234, March, 2017.

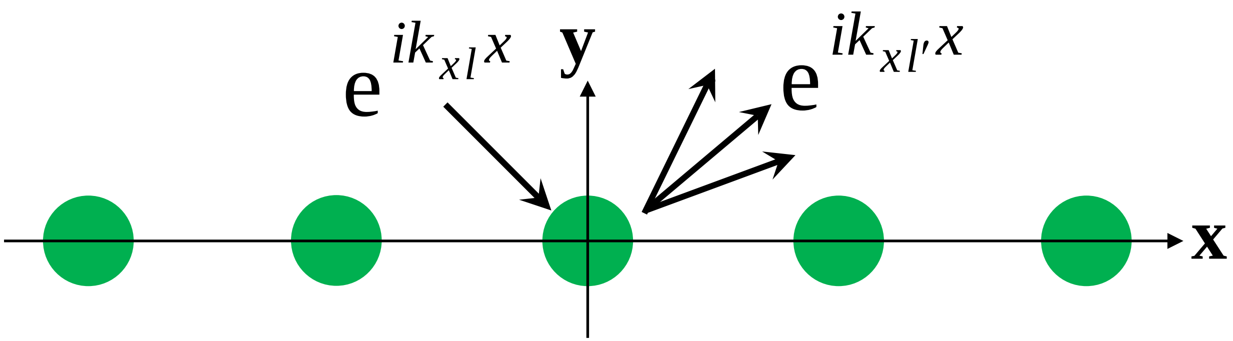
Single array

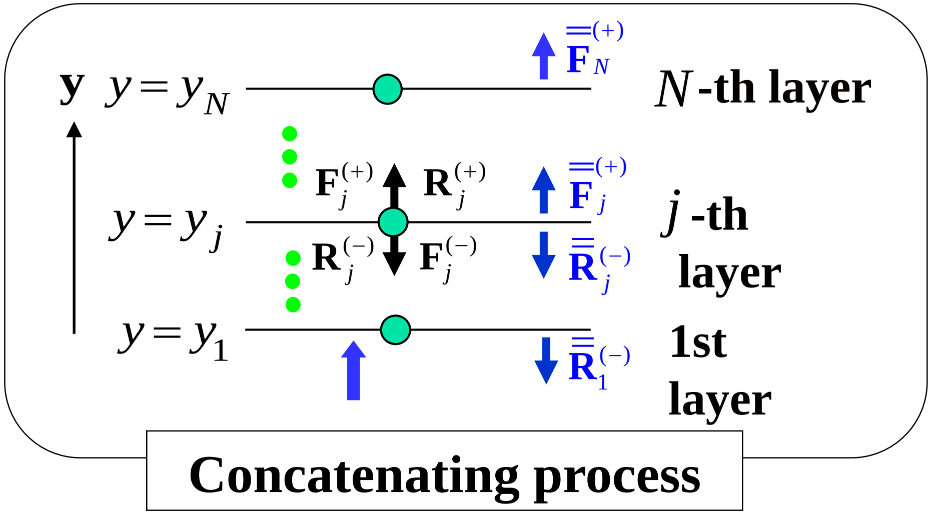
$$e^{ik_{x0}x} \Rightarrow e^{ik_{xl}x}, \quad k_{xl} = k_{x0} + \frac{2l\pi}{h}$$

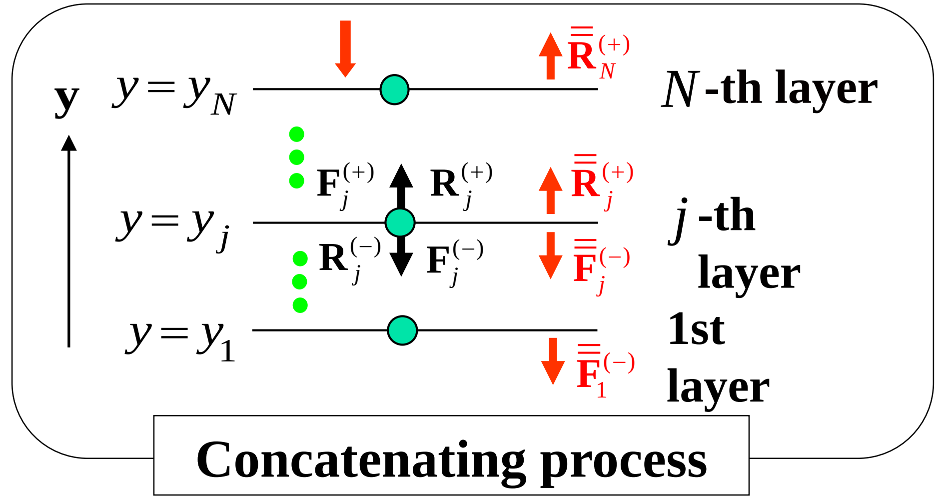


Layered array

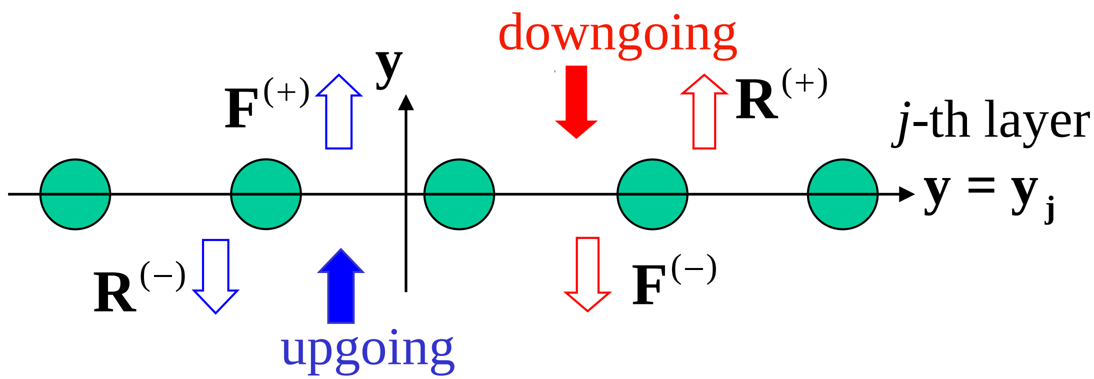
$$e^{ik_{xl}x} \Rightarrow e^{ik_{xl'}x}, \quad k_{xl'} = k_{x0} + \frac{2l'\pi}{h}$$







Reflection and Transmission matrices



$\mathbf{R}^{(\pm)}$: Reflection matrix for downgoing and upgoing space harmonics

$\mathbf{F}^{(\mp)}$: Transmission matrix for downgoing and upgoing space harmonics

➢ Formulation is general and it takes into account all Floquet modes and their interactions through the scattering by each array.

➢ Proposed approach is applied to effectively analyze the electromagnetic scattering, guidance and radiation in various configurations of the structures (periodic and/or bandgap structures) with different types and locations of the excitation sources.

1. K. Yasumoto, H. Toyama and T. Kushta, *IEEE TAP*, vol.52, pp.2603-2611, 2004.
2. V. Jandieri, K. Yasumoto and J. Pistora, *IEEE Transactions on Magnetics*, vol.53, no.4, 1000306, 2017.
3. V. Jandieri, P. Meng, K. Yasumoto and Y. Liu, *JOSA A*, vol.32, no.7, pp.1384-1389, 2015.
4. V. Jandieri, K. Yasumoto and Y. Liu, *JOSA B*, vol.29, pp.2622-2629, 2012.

Validation of the Method

Enhanced surface plasmon resonance in noble metallic systems at optical frequencies is a promising issue for realizing excellent scatterers and absorbers of the visible light.

Studies of sandwiched structures
where the metal layer that supports the surface plasmons is in contact with a dielectric slab are interesting regarding its application as a surface plasmon resonance sensor.

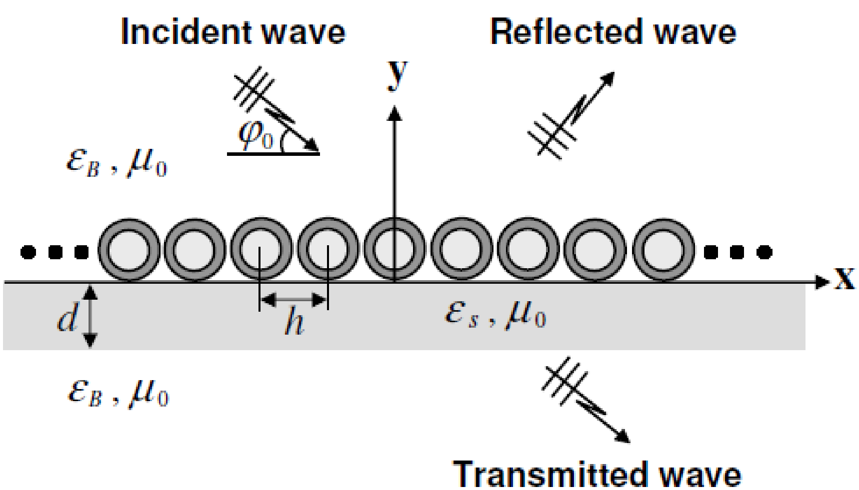
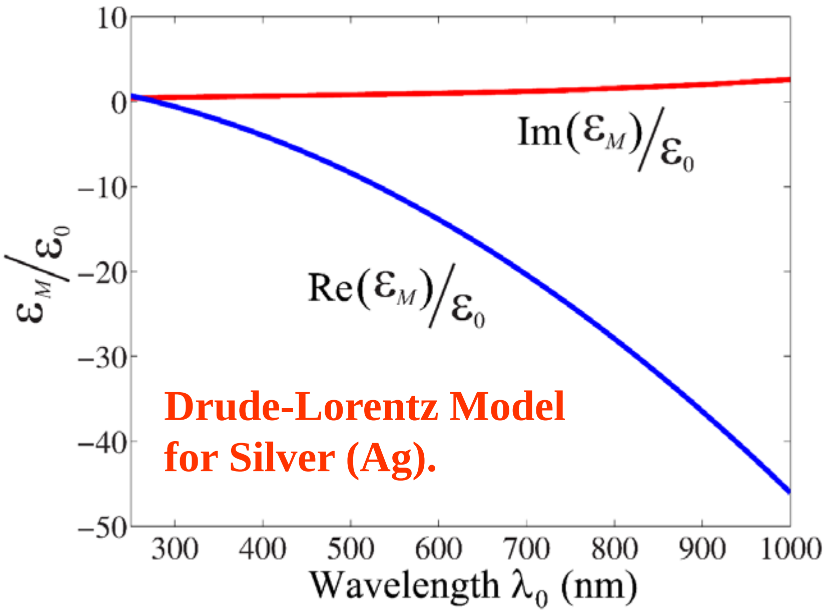


Fig. 1. Cross-sectional view of a periodic array of metal-coated nanocylinders supported on a dielectric slab and illuminated by a TE (E_x, E_y, H_z) polarized plane wave.

SCATTERING BY PLASMONIC GRATING AND ITS APPLICATION IN REFRACTIVE INDEX SENSING

$$\frac{\varepsilon_M(\omega)}{\varepsilon_0} = \varepsilon_\infty - \frac{\omega_{p,D}^2}{\omega(\omega + i\nu_D)} - \Delta_L \frac{\omega_{p,L}^2}{\omega^2 - \omega_{p,L}^2 + i\nu_L\omega}$$

ε_∞	$\omega_{p,D}/2\pi$	$\omega_{p,L}/2\pi$	$\nu_D/2\pi$	$\nu_L/2\pi$	Δ_L
3.91	13,420	6,870	84	12,340	0.76



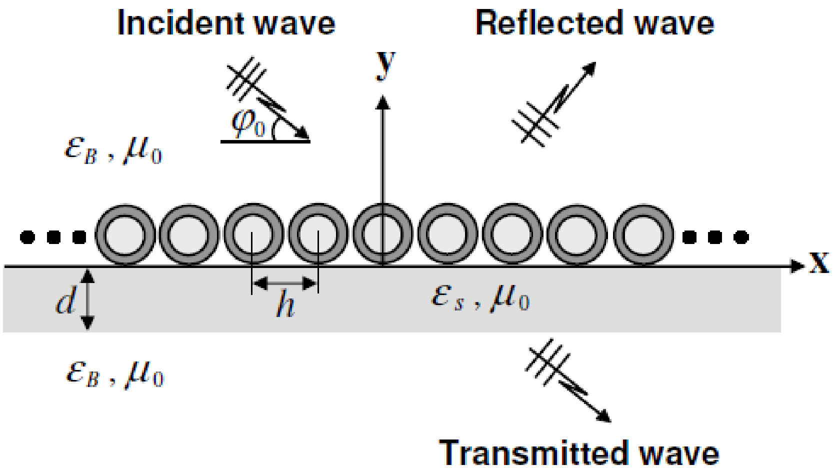
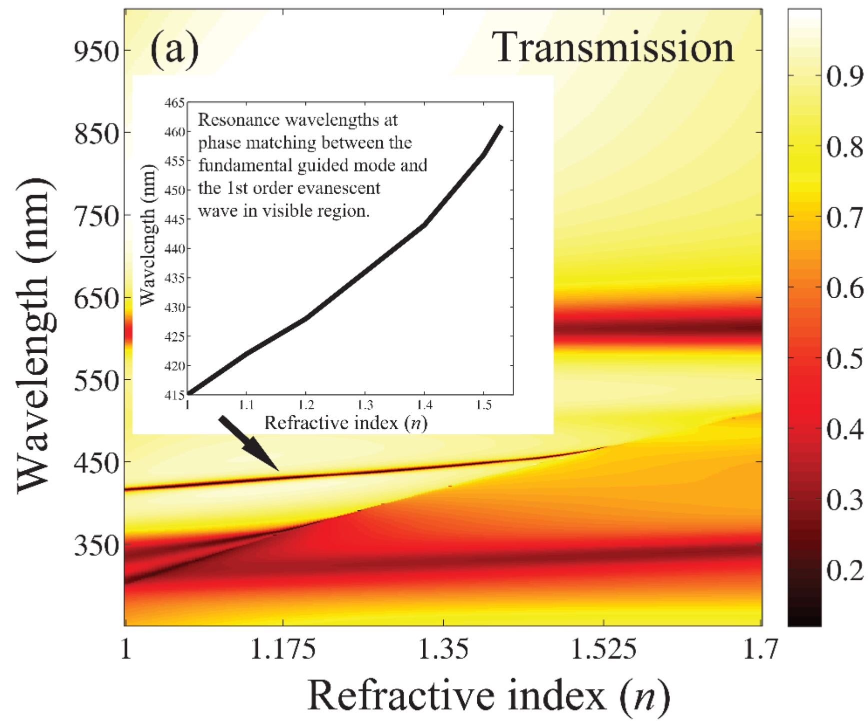
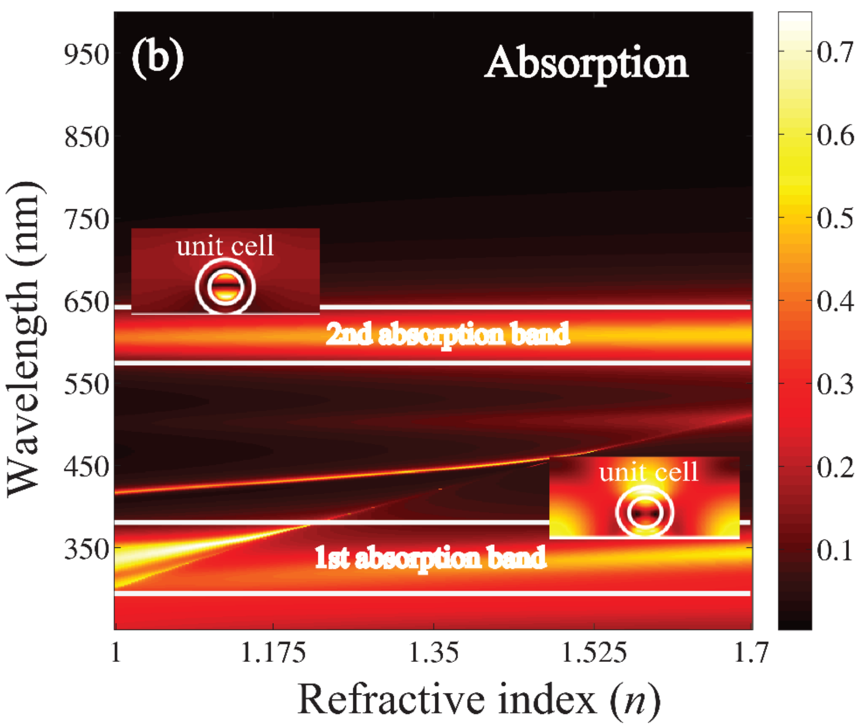


Fig. 1. Cross-sectional view of a periodic array of metal-coated nanocylinders supported on a dielectric slab and illuminated by a TE (E_x, E_y, H_z) polarized plane wave.





At one wavelength on an average PC we need approximately 0.05 s, which is nearly two orders of magnitude faster than the corresponding simulation on a frequency-domain finite-element method-(FEM) analysis with COMSOL Multiphysics needing 2 s.

Sensitivity of the structure:

$$S = \Delta\lambda / \Delta n = (\lambda_B - \lambda_{air}) / (n - 1)$$

is about 80 nm/RIU.

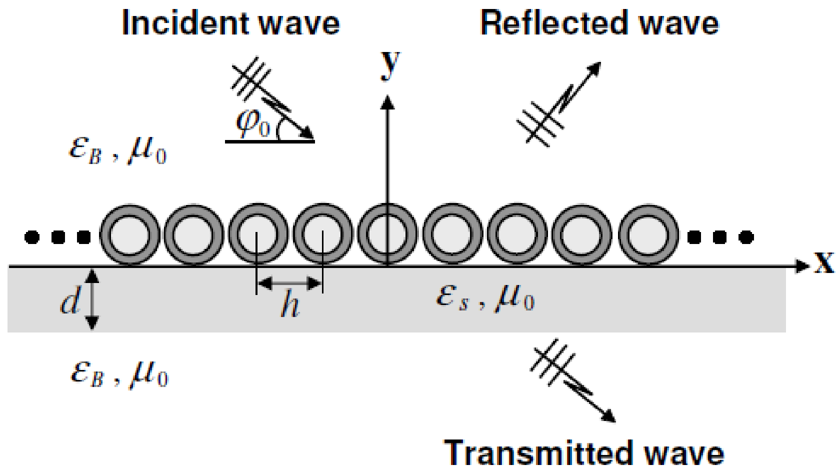
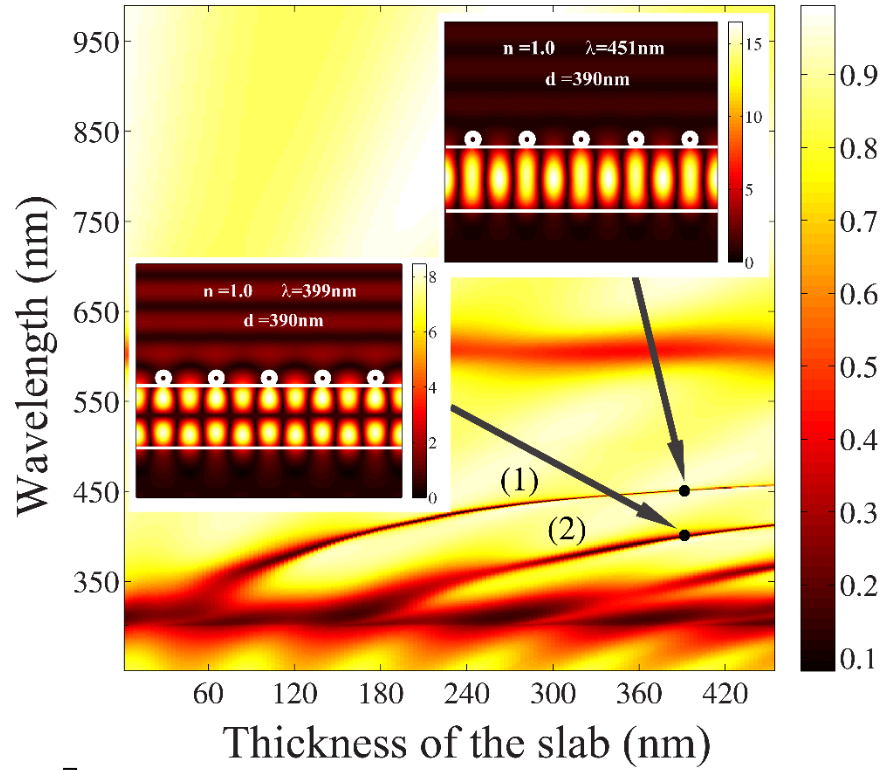


Fig. 1. Cross-sectional view of a periodic array of metal-coated nanocylinders supported on a dielectric slab and illuminated by a TE (E_x, E_y, H_z) polarized plane wave.



$$\frac{d}{\lambda}=\frac{1}{\pi\sqrt{\varepsilon_s-\left(\frac{\beta\lambda}{2\pi}\right)^2}}\left[\tan^{-1}\left(\varepsilon_s\sqrt{\frac{\left(\frac{\beta\lambda}{2\pi}\right)^2-1}{\varepsilon_s-\left(\frac{\beta\lambda}{2\pi}\right)^2}}\right)+m\frac{\pi}{2}\right]\quad (m=0,1,2,\cdots)$$

Realization of coupling between the plasmonic grating and the slab

$$\beta = k_0\cos\varphi_0+\frac{2\pi}{h}l\quad (l=\pm1,\pm2,\pm3,\cdots)$$

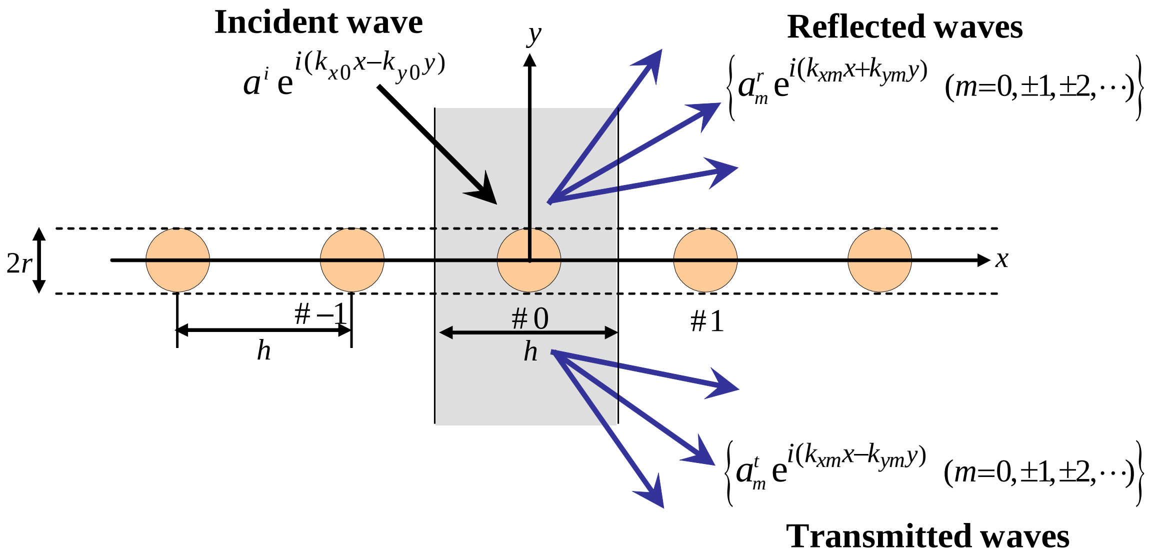


Fig. 1. Single layer of free-standing periodic array and unit cell for scattering problem

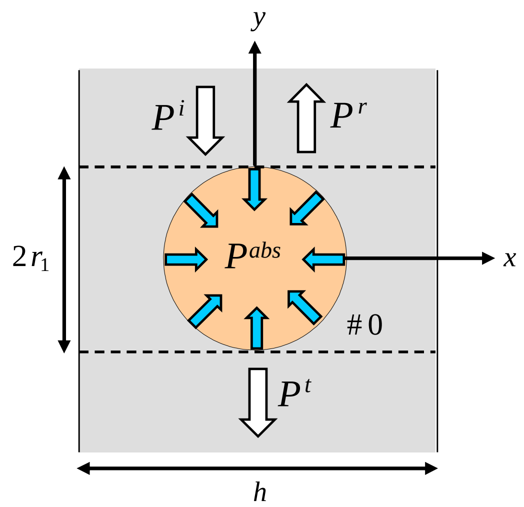


Fig. 2. Schematic view of power conservation relation over a unit cell in the x direction and a unit length in the z direction. The power conservation principle requires that the relation $P^i = P^r + P^t + P^{abs}$ should be satisfied.

It is very important to mention that the accuracy of each numerical result presented here has been tested by the energy conservation, which could be written as:

$$\sum_{\ell=-N}^{N} \left|\bar{\bar{R}}_\ell\right|^2 \frac{\mathrm{Re}\{k_{y\ell}\}}{k_{y0}} + \sum_{\ell=-N}^{N} \left|\bar{\bar{F}}_\ell\right|^2 \frac{\mathrm{Re}\{\tilde{k}_{y\ell}\}}{k_{y0}} + \frac{2\pi r}{h\sin\varphi^i} \sum_{\ell=-N}^{N} q_\ell = 1 \quad (20)$$

with

$$q_m = \mathrm{Re}\Big\{ i\,[a_{J,m} J'_m(k_0 r) + a_{H,m} H_m^{(1)\,\prime}(k_0 r)] \times [a^*_{J,m} J_m(k_0 r) + a^*_{H,m} H_m^{(1)\,*}(k_0 r)] \Big\} \quad (21)$$

where

$$\boldsymbol{a}_J = [\mathbf{I} + \mathbf{L} \cdot \bar{\bar{\mathbf{T}}}] \cdot \mathbf{P}^{(-)} \cdot \boldsymbol{a}^i \ , \quad \boldsymbol{a}_H = \bar{\bar{\mathbf{T}}} \cdot \mathbf{P}^{(-)} \cdot \boldsymbol{a}^i \quad (22)$$

Note that in comparison to the reflected and transmitted powers [the first and second LHS terms of Eq. (20)] that are calculated using the classical approach based on the space harmonic waves expansion (via Rayleigh coefficients), the absorbed power is derived by the transformation of the electromagnetic field inside the grating layer $-r \le y \le r$ from the plane wave expansion into the cylindrical wave expansion.

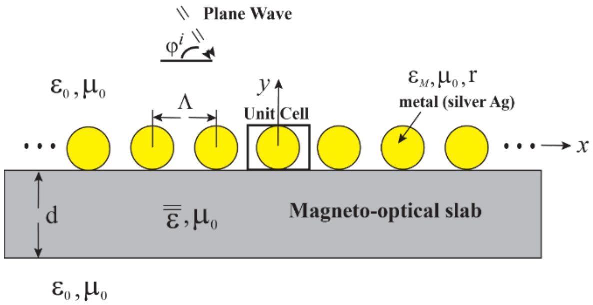
V. Jandieri, P. Meng, K. Yasumoto and Y. Liu, *JOSA A*, vol.32, no.7, pp.1384-1389, 2015.

COUPLING BETWEEN PLASMONIC GRATING AND MAGNETO-OPTICAL MEDIUM

- Studies of nanostructures with magnetic properties by optical methods are motivated by the possibilities of their applications in magnetophotonic crystals where ***functionalities that cannot be realized with isotropic materials*** (e.g. non-reciprocical propagation of optical signals, isolators, circulators, optical modulators, etc.) are possible, in recording media, in medicine, and others.
- As a magneto-optical slab we consider ***Bi-substituted gadolinium iron garnet (Bi:GIG) and measured it experimentally***. The diagonal and off-diagonal components of the dielectric permittivity tensor of the garnet (Bi:GIG) are obtained based on ellipsometric measurements.

Magneto-optical slab in case of transverse magnetization (magnetization is perpendicular to the plane of incidence) is characterized by the dielectric permittivity:

$$\overline{\overline{\varepsilon}}(\lambda)=\varepsilon_0\begin{pmatrix}\varepsilon_\perp(\lambda) & \varepsilon_\times(\lambda) & 0\\ -\varepsilon_\times(\lambda) & \varepsilon_\perp(\lambda) & 0\\ 0 & 0 & \varepsilon_\perp(\lambda)\end{pmatrix}$$



V. Jandieri, K. Yasumoto and J. Pistora, *IEEE Transactions on Magnetics*, vol.53, no.4, 1000306, 2017.

CONCLUSIONS

- A rigorous self-consistent formulation for analyzing electromagnetic scattering by a periodic grating of plasmonic nanorods is presented. The method is computationally very fast.

- It has been applied to the analysis of plasmon resonance based refractive index sensors. Such a fast and accurate method is an effective tool apt for designing and optimizing tailored sensors e.g. for advanced biomedical applications.

- Special attention has been paid to a resonance at coupling (phase-matching condition) of evanescent space harmonics of the gratings and guided mode supported by the slab. The resonance peaks sensitively depend on the refraction index of the surrounding medium, which is key to refractive index sensing applications.

- The developed self-contained approach is important for sensing the material by the spectroscopic analysis to define its unknown parameters.

Thank you very much