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Full-wave Analysis of Artificial 2D EBG Waveguides: Real and Complex Modal Solutions

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1. Extended Abstract

During the last decade, the electromagnetic properties in the microwave and optical ranges of artificial two-dimensional (2D) periodic dielectric or metallic structures, such as electromagnetic band-gap (EBG) structures and metamaterials, have been extensively studied both theoretically and experimentally. Moreover, EBG structures have been frequently employed as multifunctional and compact filters, frequency selective or polarization selective devices, and leaky-wave antennas (LWAs) [1, 2]. A periodic array of infinitely long parallel cylinders is a typical kind of discrete periodic system. When the array is multilayered, it constitutes 2D EBG structures in which any electromagnetic wave propagation is forbidden within a fairly large frequency range. The EBG waveguides can be made by removing one or more rows of the rods [2]. Knowledge of the real and complex propagation wavenumbers of bound and leaky modes supported by 2D EBG structures and waveguides is fundamental for the complete determination of both band-gap and radiative regions and for the understanding of the fundamental parameters governing the design. Therefore, the full-wave modal analysis for EBG waveguides composed by the multilayered arrays of 2D cylindrical inclusions is strictly required.

Modal propagation in 2D EBG waveguides has been extensively investigated using various approaches such as finite-difference time-domain method, the finite-difference frequency-domain method, plane-wave-expansion method [2]. These studies are concerned with the propagation features of guided modes, which are strongly confined in the guiding region of the band-gap structure and are characterized by purely real propagation wavenumbers [2]. However, different kinds of propagation regimes exist that are characterized by *complex propagation wavenumbers*. These are the cases of the stop-band regime, where the mode is still bound, but is purely reactive (no power leaks from the structure), and of the proper and improper leaky regimes, where the field confinement becomes weak and the modal field leaks out from the guiding region. The investigation of such leakage phenomena is of significant importance for the design of novel LWAs.

In this paper, a rigorous and efficient full-wave numerical approach devoted to the modal analysis of 2D EBG waveguides is presented. The proposed technique allows for the numerical study of bound and leaky modes propagating in artificial periodic structures composed by 2D cylindrical inclusions. The adopted approach uses the T-matrix and the generalized reflection and transmission matrices to characterize the nature of the cylindrical scatterer and the layered periodic structure, respectively. In this context, the efficient and accurate computation of the Lattice Sums (LSs) is required. The LSs uniquely characterize a periodic arrangement of objects and are independent of the polarization of the incident field, observation points, and the individual configuration of the scatterers. A recently developed fast and accurate calculation method for the LSs in case of *complex propagation wavenumbers* is adopted here, which resorts to original higher-order spectral and spatial Ewald representations and allows for the correct spectral determination of each spatial harmonic constituting the leaky mode [3]. Real propagation wavenumbers for bound waves in their pass-band regimes and complex propagation wavenumbers for both bound modes in their stop-band regimes and proper and improper leaky modes have been efficiently derived for typical artificial 2D EBG waveguides composed of dielectric circular rods. The results of the current method have been then compared with those obtained by means of a well-established Fourier Series Expansion method (FSEM) combined with perfectly matched layers (PMLs). An excellent agreement has been observed in all cases, while the computation time of the proposed approach, based on the LSs, was in average about 1000 time faster than that of the PML-FSEM.

2. References

1. G. V. Eleftheriades and K. G. Balmain, *Negative Refraction Metamaterials: Fundamental Principles and Applications*. Hoboken, NJ: Wiley, 2005.

2. K. Yasumoto ed., *Electromagnetic Theory and Applications for Photonic Crystals*. Boca Raton, FL: CRC Press, 2005.

3. P. Baccarelli, V. Jandieri, G. Valerio, and G. Schettini, "Efficient computation of the lattice sums for leaky waves using the Ewald method," in *Proc. 11th Eur. Conf. Antennas Propag*. (*EuCAP17*), Paris, France, 19-24 March 2017 (in press).

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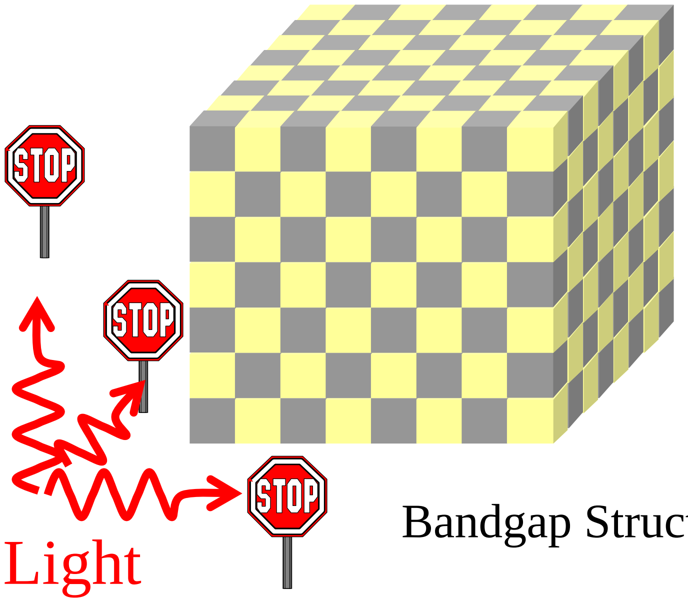
Photonic Crystals (Bandgap Structures)



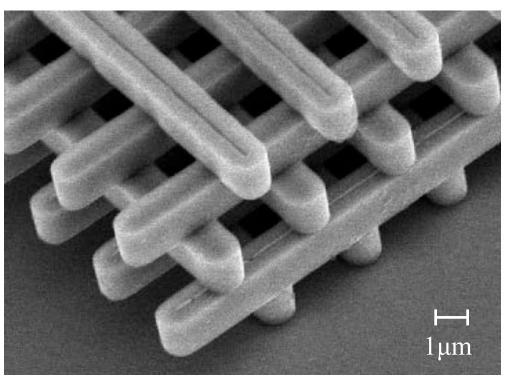
Bandgap Structures

Unique feature to localize electromagnetic waves to specific arrays and to guide along certain directions at restricted frequencies.

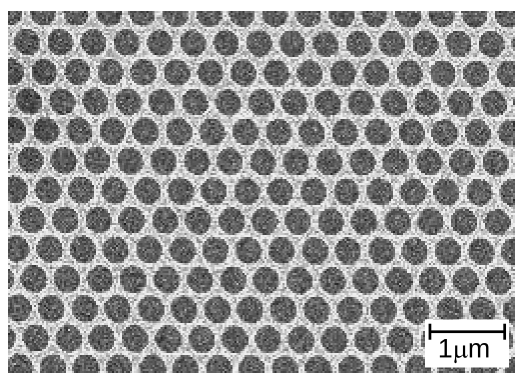
R. A. Silin and V. P. Sazonov, Slow-Wave Structures, Moscow: Soviet Radio, 1966.

E. Yablonovitch, “Inhibited spontaneous emission in solid-state physics and electronics,” Phys. Rev. Lett., vol.58, p.2059, 1987

S. John, “Strong localization of photons in certain disordered dielectric superlattices,” Phys. Rev. Lett., vol.58, p.2486, 1987



3D Woodpile type Crystals (Sandia)



2D Air-hole type Crystals (NEC)

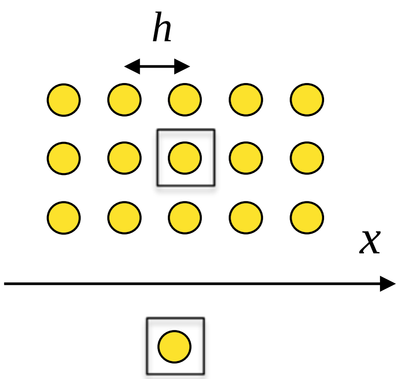
Experimental Examples

Application in:

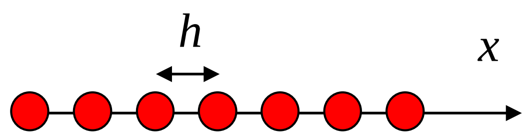
- Filters
- Waveguides
- Optical Fibers
- Antenna Substrate/Cover

Motivation (1)

2D Electromagnetic-Band-Gap (EBG) Structure

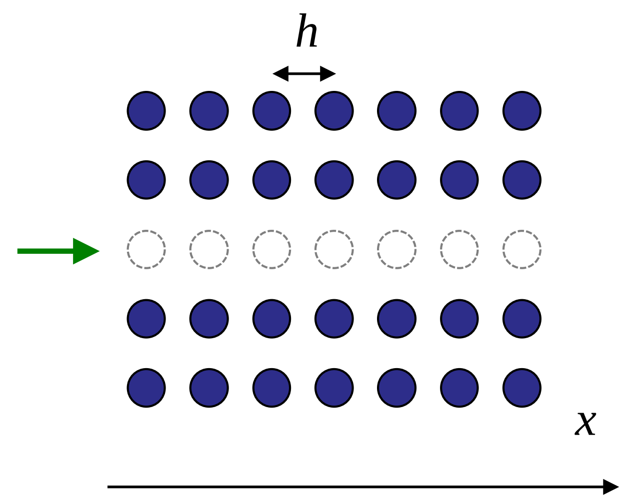


Periodic chain



Modal analysis of 2D periodic dielectric or metallic structures composed by *cylindrical inclusions* in a hosting dielectric medium

Planar 2D waveguide structure



The modal field is represented as a superposition of an *infinite* number of space harmonics

Goal: Derivation of the complex wavenumbers for *bound* modes in their stop-band regimes and *leaky* modes in their physical and non-physical regions

Space-harmonic complex wavenumber

$$k_{xn} = \beta_n + i\alpha, \quad n = 0, \pm 1, \pm 2, ..$$

$$\beta_n = \beta_0 + \frac{2\pi n}{h}$$

Space-harmonic phase constant

Modal attenuation (leakage) constant

Motivation (2)

- The complex wavenumbers are found by applying a rigorous and efficient formulation based on the Lattice Sums (LSs) technique combined with the Transition-matrix (T-matrix) approach and the recursive algorithm for the multilayered structure [1-3].

- The method is highly efficient, since the LSs are evaluated by using an effective Ewald approach [4] and a recursive relation for the layered structure is based on a simple matrix multiplication [5].

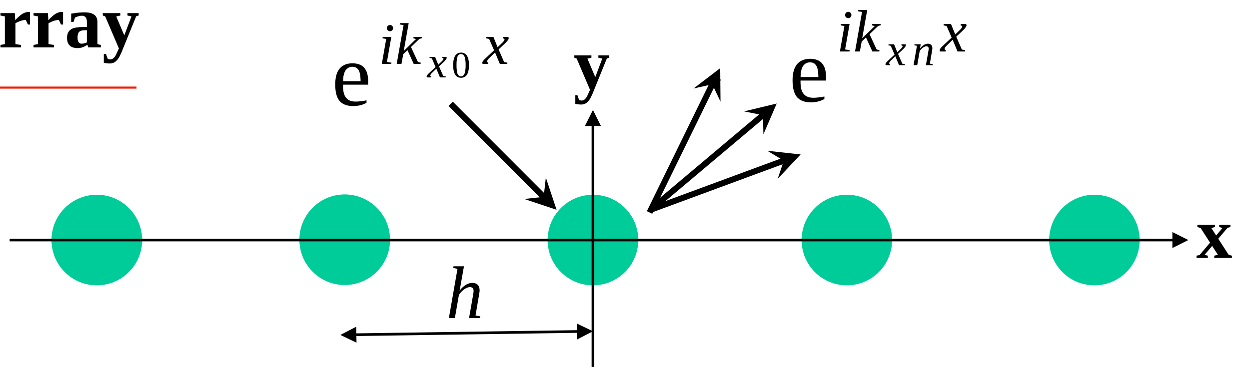
- The method allows for the appropriate choice of the spectral determination for each space harmonic in order to consider both proper and improper modal solutions.

- Radiative features of EBG Fabry-Perot cavities excited by simple localized sources (line or Hertzian dipole sources) at microwave and millimeter waves can be explained in terms of the leaky modes supported by the relevant open waveguide [6].

1. K. Yasumoto, H. Toyama and T. Kushta, *IEEE TAP*, vol.52, pp.2603-2611, 2004.
2. V. Jandieri, K. Yasumoto and J. Pistora, *IEEE Transactions on Magnetics*, vol.53, no.4, 1000306, 2017.
3. V. Jandieri, P. Meng, K. Yasumoto and Y. Liu, *JOSA A*, vol.32, no.7, pp.1384-1389, 2015.
4. P. Baccarelli, V. Jandieri, G. Valerio, and G. Schettini, *EuCAP 2017*, Paris, France, 19-24/03/2017, pp. 3222-3223.
5. W.C. Chew, *Waves and Fields in Inhomogeneous Media* (New York, Van Nostrand Reinhold, 1990).
6. S. Ceccuzzi, V. Jandieri, P. Baccarelli, C. Ponti and G. Schettini, *JOSA A*, vol.33, no.4, pp.764-770, 2016.

Formulation of the Problem (1)

Single array



$$\mathrm{e}^{ik_{x0}x} \Rightarrow \mathrm{e}^{ik_{xn}x}, \quad k_{xn} = k_{x0} + \frac{2n\pi}{h}$$

Reflected fields $(y>0)$

Transition matrix (T-matrix) — Lattice Sums

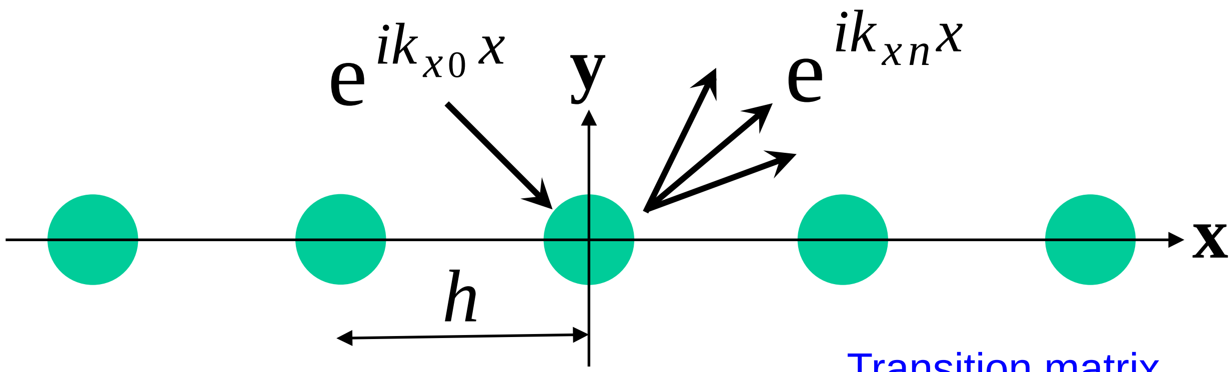
$$\psi_n^r(x,y) = r_n^{(+)} a^i \, \mathrm{e}^{i(k_{xn}x + k_{yn}y)}, \qquad r_n^{(+)} = \boldsymbol{u}_n^{(+)T} \cdot (\mathbf{I} - \mathbf{T} \cdot \mathbf{L})^{-1} \cdot \mathbf{T} \cdot \boldsymbol{p}^-$$

$r_n^{(+)}$: **0-th incident wave** ⇒ ***n*-th reflected wave**

T-matrix is obtained in a closed form for cylindrical inclusions. It is a diagonal matrix.

Formulation of the Problem (2)

Single array



Transmitted fields $(y<0)$

Transition matrix (T-matrix) Lattice Sums

$$\psi_n^t(x,y) = f_n^{(-)} a^i \, e^{i(k_{xn}x - k_{yn}y)}, \quad f_n^{(-)} = \delta_{n0} + \boldsymbol{u}_n^{(-)T} \cdot (\mathbf{I} - \mathbf{T} \cdot \mathbf{L})^{-1} \cdot \mathbf{T} \cdot \boldsymbol{p}^-$$

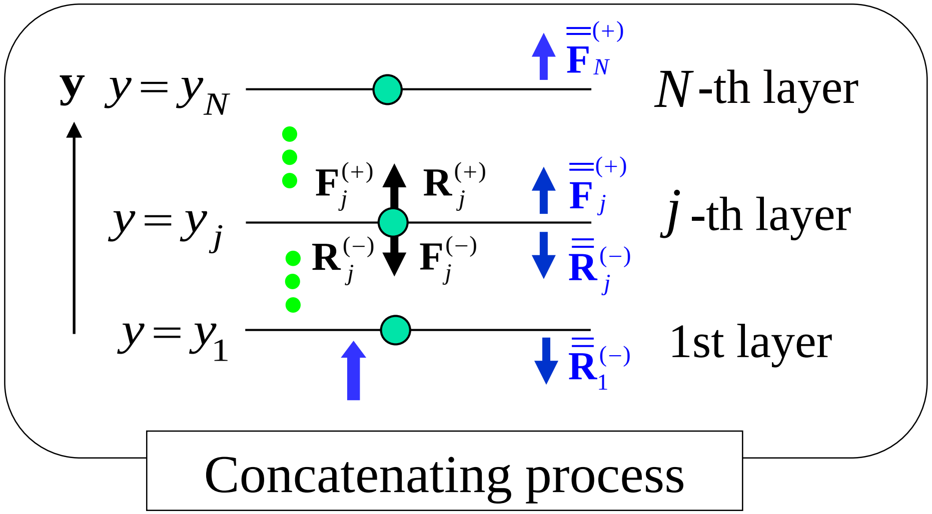
$f_n^{(-)}$: **0-th incident wave** ➡ ***n*-th transmitted wave**

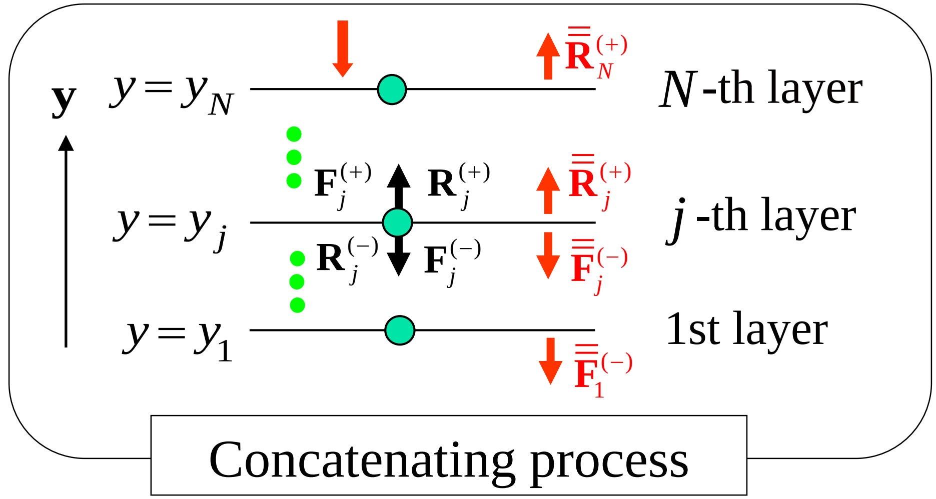
$$\boldsymbol{u}_n^{(\pm)} = [u_{nm}^{(\pm)}] = \left[\frac{2(-i)^m}{k_{yn} h} e^{\pm i m \alpha_n} \right],$$

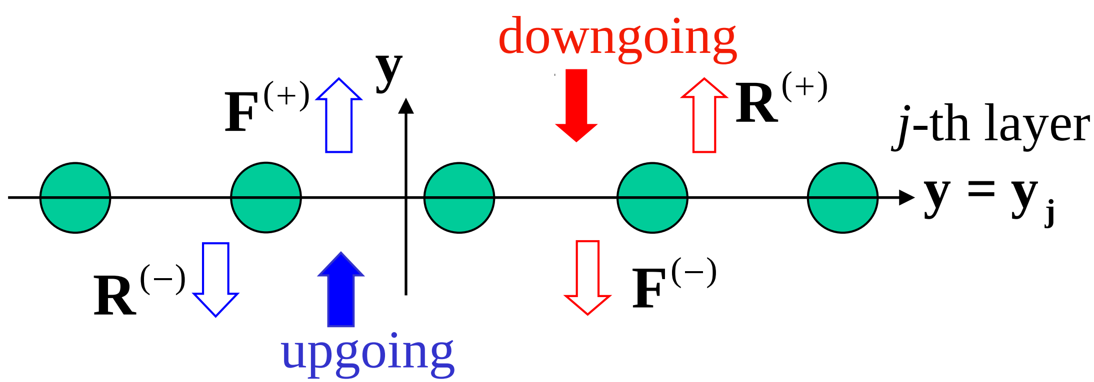
$$\alpha_n = \cos^{-1}\left(\frac{k_{xn}}{k_0}\right) \qquad k_{yn} = \sqrt{k^2 - k_{xn}^2}$$

The ***proper*** or ***improper*** features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber k_{yn}.

Formulation of the Problem (3)







$\mathbf{R}^{(\pm)}$: Reflection matrix for downgoing and upgoing space harmonics

$\mathbf{F}^{(\mp)}$: Transmission matrix for downgoing and upgoing space harmonics

Lattice-Sum (LS)

$$L_m(kh, k_{x0}h) = \sum_{n=1}^{\infty} \mathrm{H}_m^{(1)}(nkh)[\mathrm{e}^{ik_{x0}hn} + (-1)^m \, \mathrm{e}^{-ik_{x0}hn}]$$

L_m **depends only on** k**,** h **(i.e., the period), and** k_{x0} **(i.e., the fundamental space-harmonic wavenumber); It is independent of the polarization of the incident field. It uniquely characterizes a periodic array of sources.**

$$k_{x0} = \beta_0$$

Real wavenumber [1, 2]:
Slow converging series

$$k_{x0} = \beta_0 + i\alpha$$

Complex wavenumber [3]:
Exponentially diverging series

[1] K. Yasumoto and K. Yoshitomo, *IEEE TAP*, vol.47, pp.1050-1055, 1999.

[2] C. M. Linton, “Lattice sums for the Helmholtz equation,” SIAM Review, vol. 52, no. 4, pp. 630-674, 2010.

[3] P. Baccarelli, V. Jandieri, G. Valerio and G. Schettini, “Efficient computation of the lattice sums for leaky waves using the Ewald method,” *Proceedings of the 11-th European Conference on Antennas and Propagation*, Paris, France, pp. 3233-3234, March, 2017.

LSs for the complex wavenumber can be accurately calculated using *Ewald method*.
We calculate separately spectral and spatial series:

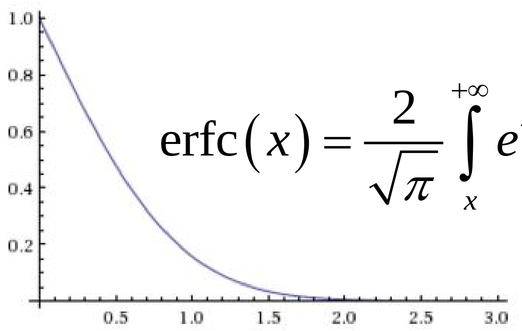
$$L_m = L_m^{E_{spectral}}(kh, k_{x0}h) + L_m^{E_{spatial}}(kh, k_{x0}h)$$

Lattice-Sum Ewald Spectral Series

After several mathematical manipulations, for the **Spectral** series we finally obtain:

$$L_m^{E_{spectral}}(kh, k_{x0}h) = \frac{2i^m}{h} \sum_{n=-\infty}^{\infty} \left(\frac{k_{xn}}{k}\right)^m \sum_{q=0}^{[m/2]} (-1)^q \binom{m}{2q} \left(\frac{k_{yn}}{k_{xn}}\right)^{2q} C_{q,n}, \quad m \geq 0$$

$$C_{q,n} = \frac{1}{k_{yn}} \operatorname{erfc}\left(-i\frac{hk_{yn}}{2E_{spl}}\right) - \frac{e^{\left(\frac{hk_{yn}}{2E_{spl}}\right)^2}}{k_{yn}} \sum_{s=1}^{q} \frac{\left(-i\frac{hk_{yn}}{2E_{spl}}\right)^{1-2s}}{\Gamma\left(\frac{3}{2}-s\right)},$$



$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{+\infty} e^{-t^2} \mathrm{d}t$$

$$k_{yn} = \sqrt{k^2 - k_{xn}^2}$$

The ***spectral*** higher-order Ewald series presents a ***very fast Gaussian convergence*** also for **complex** k_{x0}

The ***proper*** or ***improper*** features can be easily determined by imposing the appropriate square root determination for the vertical wavenumber k_{yn}.

Lattice-Sum Ewald Spatial Series (1)

For the **Spatial** series, we obtain:

$$L_m^{E_{spatial}}(kh, k_{x0}h) = \delta_{m,0}\left[-1 - \frac{i}{\pi}\mathrm{Ei}\left(\frac{k^2h^2}{4E_{spl}^2}\right)\right]$$

$$+\frac{2^{m+1}}{i\pi}\sum_{n=1}^{\infty}[e^{ink_{x0}h} + (-1)^m e^{-ink_{x0}h}]\left(\frac{n}{kh}\right)^m \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2 + \frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

How to compute this integral? A numerical integration could be slow and not robust…

$$I_m = \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2 + \frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

Lattice-Sum Ewald Spatial Series (2)

$$I_m = \int_{E_{spl}}^{\infty} \frac{e^{-n^2\eta^2 + \frac{k^2h^2}{4\eta^2}}}{\eta^{-2m+1}} d\eta, \quad m \geq 0,$$

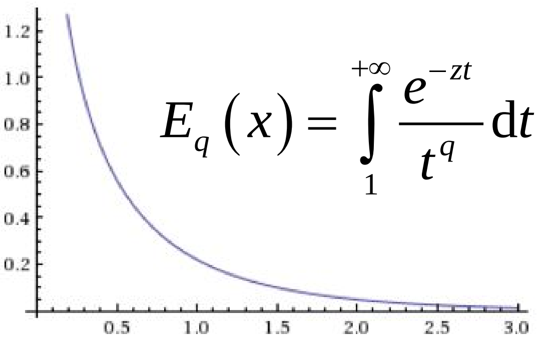
A recurrence relation in *m* have been obtained to **significantly speed up** the evaluation of the integrals in the spatial Ewald series

$$I_{m+1} = \frac{1}{2n^2}\left(2mI_m - \frac{k^2h^2}{2} I_{m-1} + E_{spl}^{2m} e^{-n^2E_{spl}^2} e^{k^2h^2/4E_{spl}^2}\right)$$

I_0 and I_1 can be easily obtained as

$$I_0 = \frac{1}{2}\sum_{p=0}^{\infty}\left(\frac{kh}{2E_{spl}}\right)^{2p} \frac{1}{p!} E_{p+1}\left(n^2E_{spl}^2\right)$$

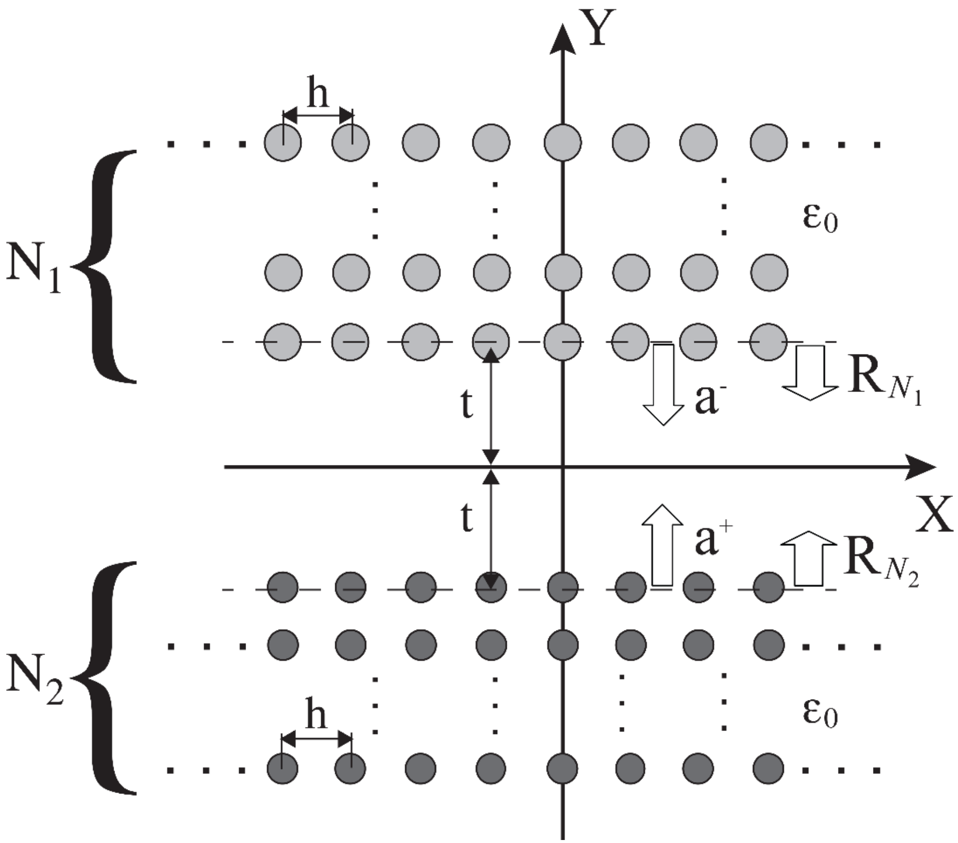
$$I_1 = \frac{E_{spl}^2}{2}\left[\frac{1}{n^2E_{spl}^2} e^{-n^2E_{spl}^2} + \sum_{p=1}^{\infty}\left(\frac{kh}{2}\right)^{2p} \frac{1}{p!}\frac{1}{E_{spl}^{2p}} E_p\left(n^2E_{spl}^2\right)\right]$$



$$E_q(x) = \int_1^{+\infty} \frac{e^{-zt}}{t^q} \mathrm{d}t$$

The spatial higher-order Ewald series also presents a ***very fast Gaussian convergence*** also for **complex** k_{x0}.

Formulation of the Problem (4)



$$\mathbf{a}^{-}(y=t)=\mathbf{R}_{N_1}(k_{x0})\mathbf{a}^{+}(y=t)$$

$$\mathbf{a}^{+}(y=-t)=\mathbf{R}_{N_2}(k_{x0})\mathbf{a}^{-}(y=-t)$$

$$\mathbf{a}^{+}(y=t)=\mathbf{D}(k_{x0})\mathbf{a}^{+}(y=-t)$$

$$\mathbf{a}^{-}(y=-t)=\mathbf{D}(k_{x0})\mathbf{a}^{-}(y=t)$$

$$\mathbf{D}(k_{x0})=\left[e^{ik_{yn}2t}\delta_{ll'}\right]$$

Dispersion Equation:

$$\det\left[\mathbf{I}-\mathbf{D}(k_{x0})\cdot\mathbf{R}_{N_1}(k_{x0})\cdot\mathbf{D}(k_{x0})\cdot\mathbf{R}_{N_2}(k_{x0})\right]=0$$

Spectral Properties of the Modal Solution

Behavior of each space harmonic in the air region, e.g.: $y > 0,\ e^{ik_{yn}y}$

$$k_{yn} = \sqrt{k^2 - k_{xn}^2} = \sqrt{k^2 - (\beta_n + i\alpha)^2} = \beta_{yn} + i\alpha_{yn}$$

Exponentially decaying

Exponentially growing

Proper determination: $\mathrm{Im}\{k_{yn}\} > 0$

Improper determination: $\mathrm{Im}\{k_{yn}\} < 0$

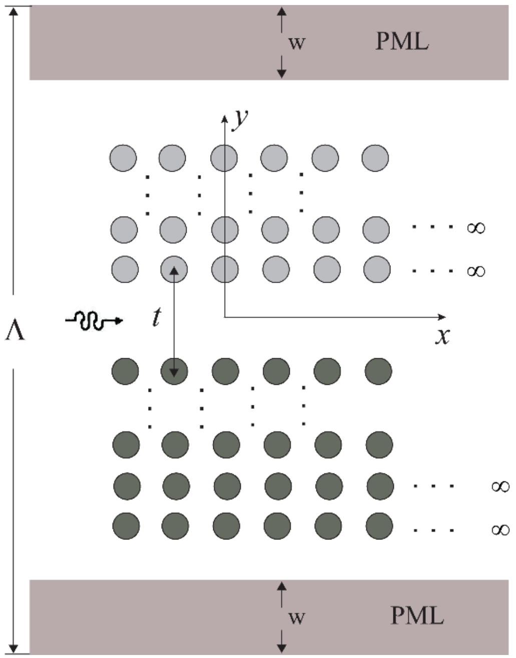
Physical choice for each space harmonic

	$\alpha = 0$	$\beta_n \alpha > 0$	$\beta_n \alpha < 0$
	Surface Wave *(proper)*	**Forward Wave** *(improper leaky wave)*	**Backward Wave** (proper leaky wave)
Slow harmonics:	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$	$\lvert\beta_n\rvert > k : \mathrm{Im}\{k_{yn}\} > 0$
Fast harmonics:		$\lvert\beta_n\rvert < k : \mathrm{Im}\{k_{yn}\} < 0$	$\lvert\beta_n\rvert < k : \mathrm{Im}\{k_{yn}\} > 0$

Full-Wave Modal Analysis. Numerical Results

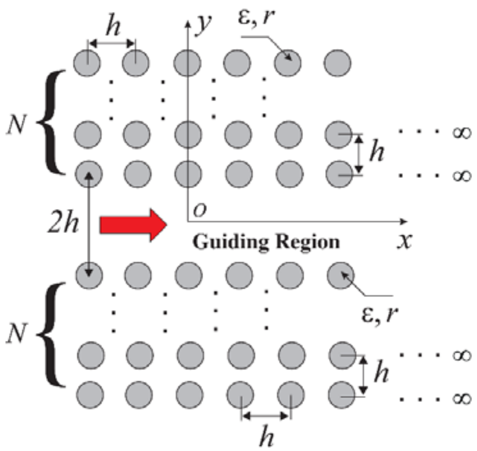
Fourier Series Expansion Method (FSEM) Combined with Perfectly Matched Layers (PMLs)

- For validation purposes, a Fourier Series Expansion method (FSEM) with perfectly matched layers (PMLs) has been implemented to analyze 2-D EBG waveguides composed by cylindrical inclusions, whose section can have an arbitrary geometry.
- The electric and magnetic fields are approximated by truncated Fourier series.
- The FSEM uses the staircase approximation of the circular section by applying several multilayered thin rectangular strips.
- A substantial number of numerical tests are required to properly choose the PML parameters in order to distinguish the leakage loss from the material loss caused by the assumed conductivity in the PMLs.



D. Zhang and H. Jia, "Numerical analysis of leaky modes in two-dimensional photonic crystal waveguides using Fourier series expansion method with perfectly matched layer," *IEICE Transactions on Electronics*, vol. E90-C, pp. 613-622, 2007.

W1 Type EBG Waveguide: Improper Leaky Mode (1)



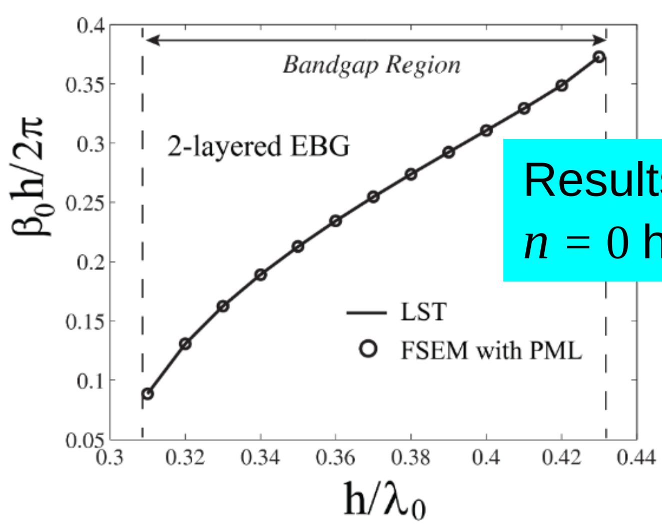
$N = 2$

$\varepsilon = 11.9\ \varepsilon_0$

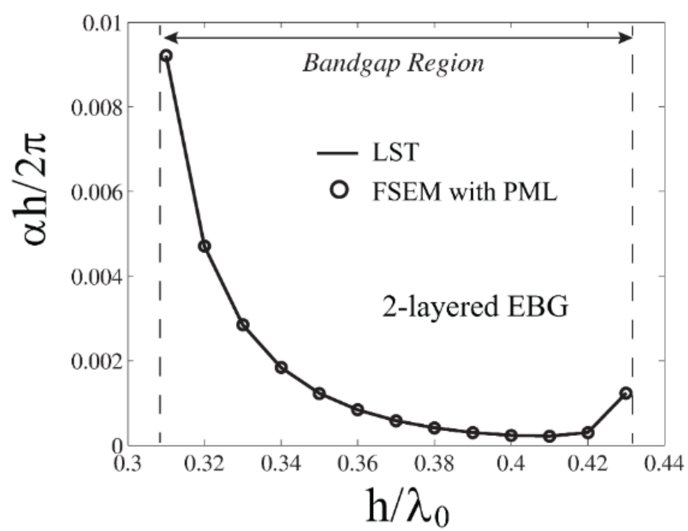
$r = 0.2\ h$

The periodic array of dielectric cylindrical rods has a bandgap region in the normalized frequency range $0.303 < h/\lambda_0 < 0.432$

Lowest order TE leaky mode (E_z, H_x, H_y)



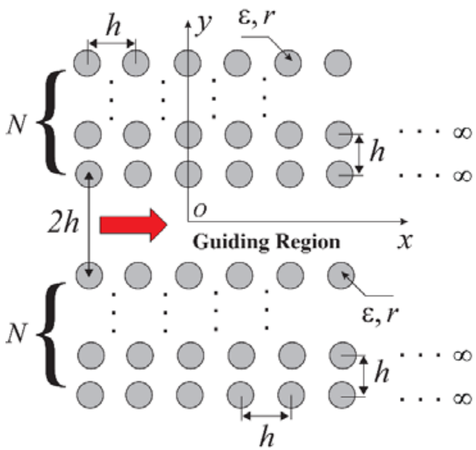
Results for
n = 0 harmonic



The $n = 0$ space harmonic is fast and has an improper determination in the LST, whereas all other harmonics are proper

The results obtained with the LST and FSEM with PML are in very close agreement with an accuracy of at least four digits

W1 Type EBG Waveguide: Improper Leaky Mode (2)



$N = 3$

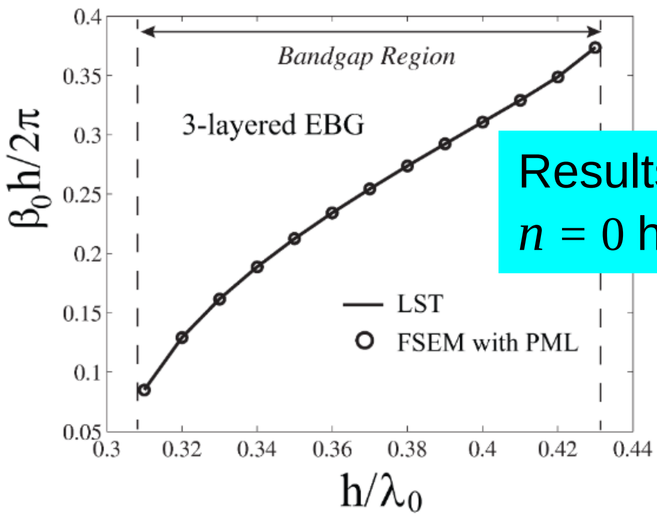
$\varepsilon = 11.9\ \varepsilon_0$

$r = 0.2\ h$

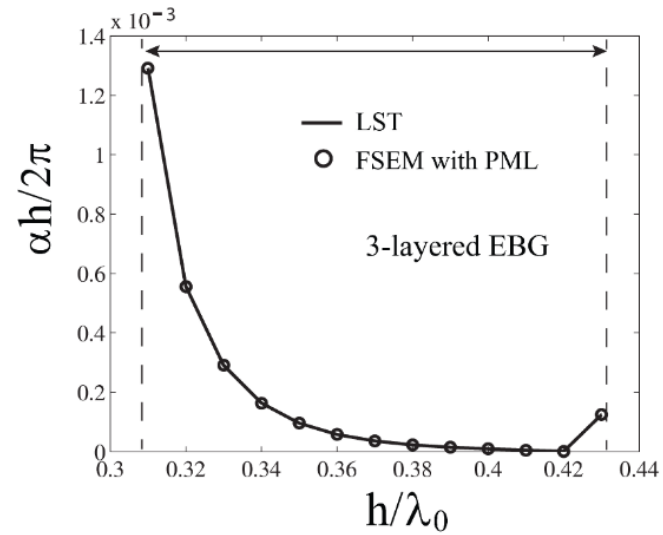
When the number of the EBG layers is increasing the attenuation constant substantially decreases.

$n = 0$ space harmonic is fast and has an improper determination in the LST

Lowest order TE leaky mode (E_z, H_x, H_y)

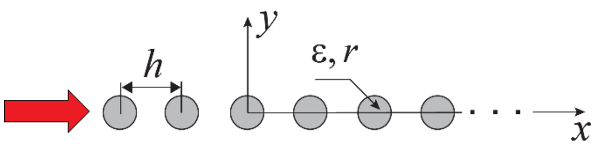


Results for
$n = 0$ harmonic



The LST is more efficient than the FSEM with PML: 0.02 s against 20 s per one frequency point with the same 3.6 GHz Intel Core i7 with 8 GB RAM

Periodic Chain of Dielectric Circular Rods



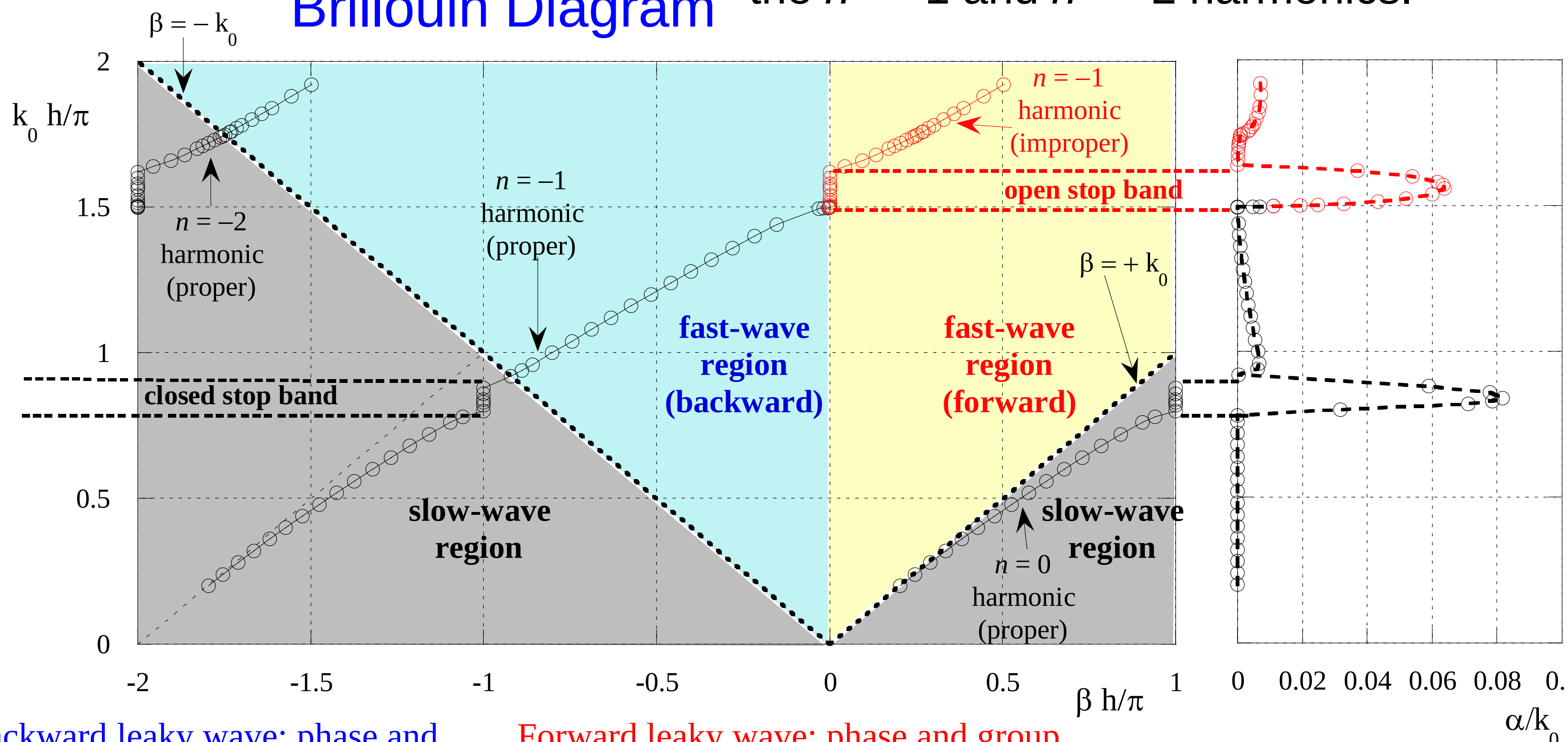
$\varepsilon = 2.25\varepsilon_0$

$r = 0.4167\ h$

Lowest order
TE leaky mode
(E_z, H_x, H_y)

Complete Brillouin diagram with the details of the backward and forward fast-wave regions for the $n = -1$ harmonic (proper/improper determinations), the closed and open stop-band regions, and the grating lobe due to the simultaneous radiation from the $n = -1$ and $n = -2$ harmonics.

Brillouin Diagram



Backward leaky wave: phase and group velocities of opposite signs

Forward leaky wave: phase and group velocities are in the same direction

Conclusions

- A full-wave numerical approach for the analysis of modes with complex propagation wavenumber in periodic and bandgap structures composed of 2D cylindrical inclusions has been proposed.
- The method is based on the lattice sums (LSs) technique and has been suitably adapted to the analysis of modes with complex propagation wavenumbers, by applying higher-order Ewald representation, in terms of spectral and spatial series having Gaussian convergence.
- All the possible bound and leaky modes propagating along periodic and bandgap structures composed of 2D cylindrical inclusions can be considered.
- An exhaustive analysis of two reference 2D EBG waveguides has allowed us to characterize the relevant Pass-Band and Band-Gap Zones and the Radiative Regions.

Future works:

- Analysis of leakage and radiative phenomena in 2D EBG structures
- Design of filters and periodic Leaky Wave Antennas based on EBG structures

Thank You!