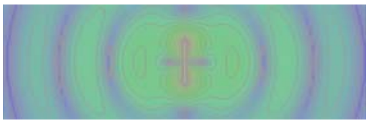
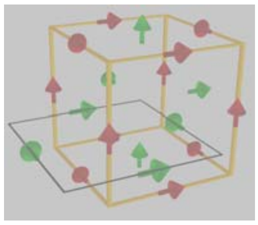


FDTD Method

$$\mu \frac{\partial}{\partial t}\vec{H} = -\nabla \times \vec{E}$$

$$\varepsilon \frac{\partial}{\partial t}\vec{E} = \nabla \times \vec{H} - \vec{J}$$



IMST

Outline

- Basics
  - Maxwell's Equations
  - Spatial discretisation/Time discretisation
  - Equivalent Circuit for FDTD
  - Stability
  - Ports
- Simple Examples
- Accuracy and Losses
- Speedups on modern Computers
- Conclusions

IMST

Maxwell‘s Equations

$$\mu \frac{\partial}{\partial t} \vec{H} = -\nabla \times \vec{E}$$

$$\varepsilon \frac{\partial}{\partial t} \vec{E} = \nabla \times \vec{H} - \vec{J}$$

Parameters

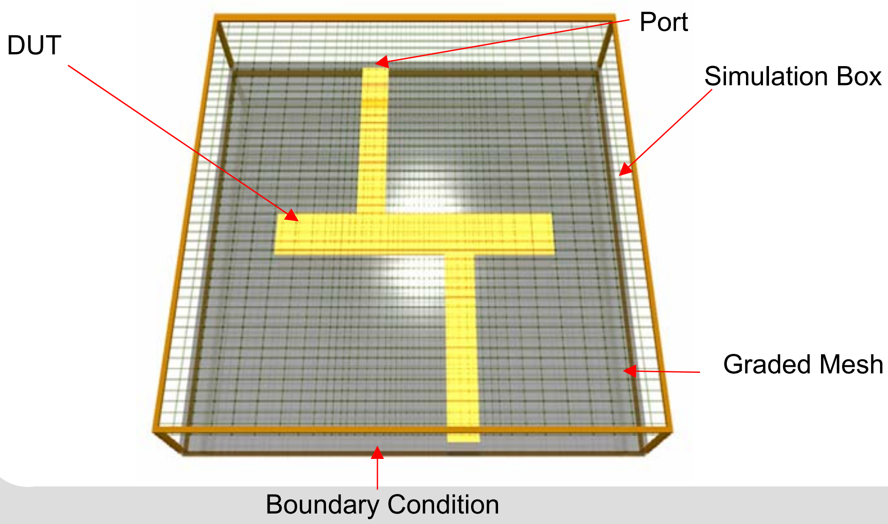
Electric Field

Magnetic Field

Excitation

- Hyperbolic partial differential equation, initial boundary value problem
- Time domain tracking of the electromagetic field
- Passive component analysis

Spatial FDTD Principle



FDTD Basics

Maxwell´s Equations

$$\mu\frac{\partial}{\partial t}\vec{H} = -\nabla\times\vec{E}$$

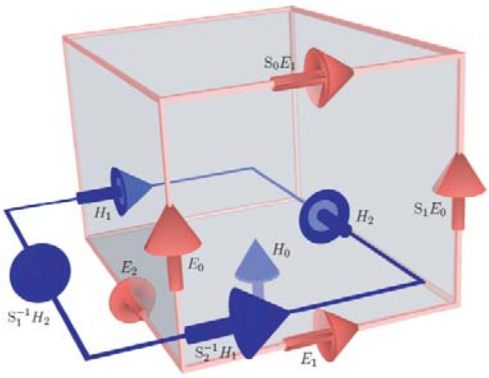
$$\varepsilon\frac{\partial}{\partial t}\vec{E} = \nabla\times\vec{H}-\vec{J}$$

Spatial discretization

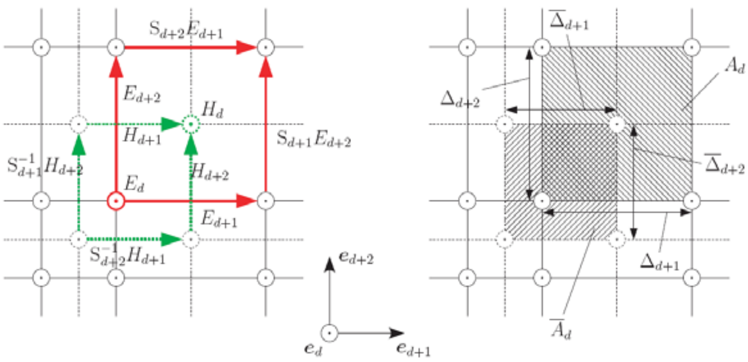
$$\mu\frac{\partial}{\partial t}\vec{H} = -\frac{1}{\Delta_x}\sum_{\circlearrowright}\vec{E}$$

$$\varepsilon\frac{\partial}{\partial t}\vec{E} = \frac{1}{\Delta_x}\sum_{\circlearrowright}\vec{H}-\vec{J}$$

Yee cell



Spatial Discretisation



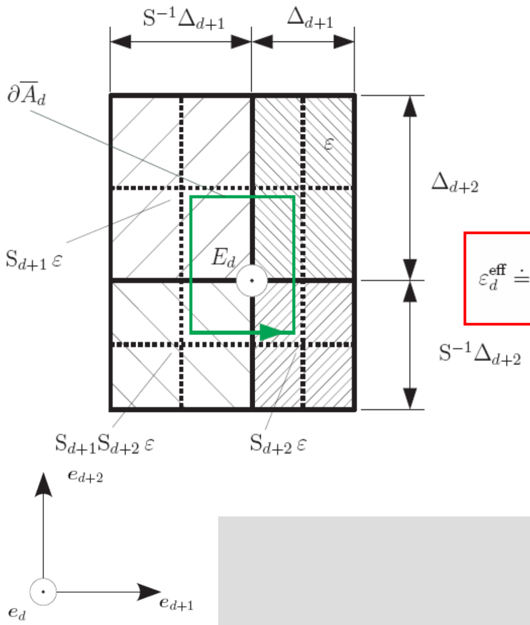
$$\oint_{\partial\overline{A}_d}\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{\ell}\approx\{1-\mathrm{S}_{d+1}^{-1}\}\,H_{d+2}\,\overline{\Delta}_{d+2}-\{1-\mathrm{S}_{d+2}^{-1}\}\,H_{d+1}\,\overline{\Delta}_{d+1}=$$

$$+\left(\varepsilon_d^{\mathrm{eff}}\frac{\mathrm{d}}{\mathrm{d}t}E_d+\kappa_d^{\mathrm{eff}}E_d-J_d^{\mathrm{src}}\right)\overline{\Delta}_{d+1}\overline{\Delta}_{d+2}\approx\iint_{\overline{A}_d}\ldots\cdot\mathrm{d}\boldsymbol{A},$$

$$\oint_{\partial A_d}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{\ell}\approx\{\mathrm{S}_{d+1}-1\}\,E_{d+2}\,\Delta_{d+2}-\{\mathrm{S}_{d+2}-1\}E_{d+1}\,\Delta_{d+1}=$$

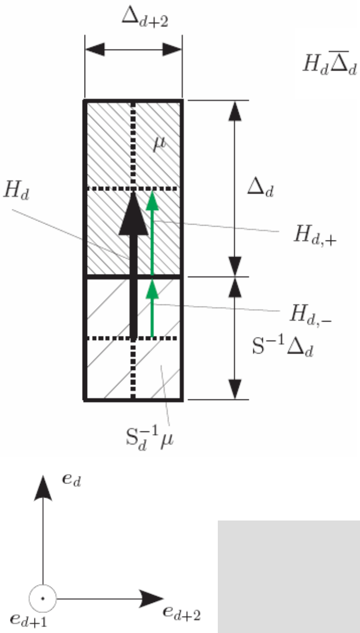
$$-\left(\mu_d^{\mathrm{eff}}\frac{\mathrm{d}}{\mathrm{d}t}H_d+\sigma_d^{\mathrm{eff}}H_d\right)\Delta_{d+1}\Delta_{d+2}\approx\iint_{A_d}\ldots\cdot\mathrm{d}\boldsymbol{A}.$$

Effective Electric Materials



$$\varepsilon_d^{\text{eff}} \doteq \frac{\frac{1}{4}\left\{1+\mathrm{S}_{d+1}^{-1}+\mathrm{S}_{d+2}^{-1}+\mathrm{S}_{d+1}^{-1}\mathrm{S}_{d+2}^{-1}\right\}\left[\varepsilon\,\Delta_{d+1}\,\Delta_{d+2}\right]}{\frac{1}{4}\left\{1+\mathrm{S}_{d+1}^{-1}+\mathrm{S}_{d+2}^{-1}+\mathrm{S}_{d+1}^{-1}\mathrm{S}_{d+2}^{-1}\right\}\left[\Delta_{d+1}\,\Delta_{d+2}\right]}.$$

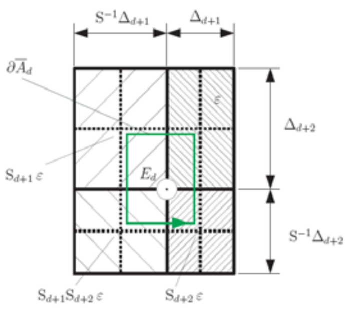
Effective Magnetic Materials



$$H_d\overline{\Delta}_d = H_{d,+}\frac{1}{2}\Delta_d + H_{d,-}\frac{1}{2}\mathrm{S}_d^{-1}\Delta_d = B_d\frac{1}{2}\left\{1+\mathrm{S}_d^{-1}\right\}\left[\frac{\Delta_d}{\mu}\right] \doteq B_d\frac{\overline{\Delta}_d}{\mu_d^{\text{eff}}},$$

$$\mu_d^{\text{eff}} \doteq \frac{\overline{\Delta}_d}{\frac{1}{2}\left\{1+\mathrm{S}_d^{-1}\right\}\left[\frac{\Delta_d}{\mu}\right]} = \frac{\frac{1}{2}\left\{1+\mathrm{S}_d^{-1}\right\}\Delta_d}{\frac{1}{2}\left\{1+\mathrm{S}_d^{-1}\right\}\left[\frac{\Delta_d}{\mu}\right]},$$

Electric Equivalent Circuit Elements



$$\{1-\mathrm{S}_{d+1}^{-1}\} H_{d+2}\overline{\Delta}_{d+2} - \{1-\mathrm{S}_{d+2}^{-1}\} H_{d+1}\overline{\Delta}_{d+1} = -\overline{\Delta}_{d+1}\overline{\Delta}_{d+2} J_d^{\mathrm{src}}$$

$$+\frac{1}{4}\{1+\mathrm{S}_{d+1}^{-1}+\mathrm{S}_{d+2}^{-1}+\mathrm{S}_{d+1}^{-1}\mathrm{S}_{d+2}^{-1}\}\left[\varepsilon\,\Delta_{d+1}\,\Delta_{d+2}\right]\frac{1}{\Delta_d}\frac{\mathrm{d}}{\mathrm{d}t}E_d\Delta_d$$

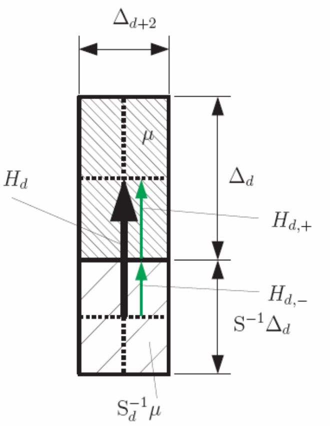
$$+\frac{1}{4}\{1+\mathrm{S}_{d+1}^{-1}+\mathrm{S}_{d+2}^{-1}+\mathrm{S}_{d+1}^{-1}\mathrm{S}_{d+2}^{-1}\}\left[\kappa\,\Delta_{d+1}\,\Delta_{d+2}\right]\frac{1}{\Delta_d}E_d\Delta_d,$$

$$E_d\,\Delta_d \doteq v_d \quad \Downarrow \quad H_d\,\overline{\Delta}_d \doteq i_d$$

$$\{1-\mathrm{S}_{d+1}^{-1}\}\, i_{d+2} - \{1-\mathrm{S}_{d+2}^{-1}\}\, i_{d+1} + i_{\mathrm{src},d} = +C_d\frac{\mathrm{d}}{\mathrm{d}t}v_d + G_d v_d,$$

- Kirchhoffsche Knotenregel

Magnetic Equivalent Circuit Elements



$$\{\mathrm{S}_{d+1}-1\}\, E_{d+2}\,\Delta_{d+2} - \{\mathrm{S}_{d+2}-1\}E_{d+1}\,\Delta_{d+1} =$$

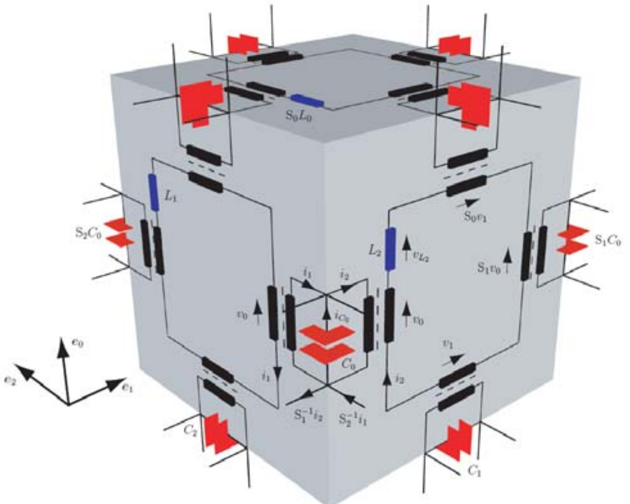
$$-\frac{\Delta_{d+1}\Delta_{d+2}}{\frac{1}{2}\{1+\mathrm{S}_d^{-1}\}\left[\frac{\Delta_d}{\mu}\right]}\frac{\mathrm{d}}{\mathrm{d}t}H_d\overline{\Delta}_d + \frac{\Delta_{d+1}\Delta_{d+2}}{\frac{1}{2}\{1+\mathrm{S}_d^{-1}\}\left[\frac{\Delta_d}{\sigma}\right]}H_d\overline{\Delta}_d.$$

$$E_d\,\Delta_d \doteq v_d \quad \Downarrow \quad H_d\,\overline{\Delta}_d \doteq i_d$$

$$\{\mathrm{S}_{d+1}-1\}\, v_{d+2} - \{\mathrm{S}_{d+2}-1\}\, v_{d+1} = -L_d\frac{\mathrm{d}}{\mathrm{d}t}i_d - R_d i_d,$$

- Kirchhoffsche Maschenregel

3D Equivalent Circuit for FDTD Method

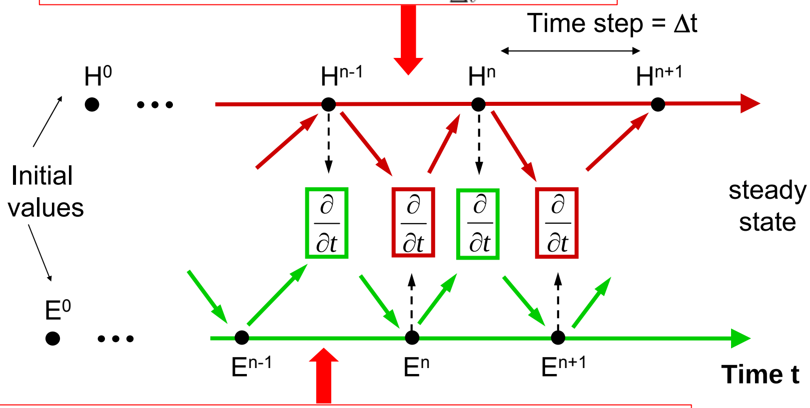


$$\{1 - S_{d+1}^{-1}\}\, i_{d+2} - \{1 - S_{d+2}^{-1}\}\, i_{d+1} + i_{\mathrm{src},d} = + C_d \frac{\mathrm{d}}{\mathrm{d}t} v_d + G_d\, v_d,$$

$$\{S_{d+1} - 1\}\, v_{d+2} - \{S_{d+2} - 1\}\, v_{d+1} = - L_d \frac{\mathrm{d}}{\mathrm{d}t} i_d - R_d\, i_d,$$

Time Discretisation

$$\{S_{d+1} - 1\}\, v_{d+2} - \{S_{d+2} - 1\}\, v_{d+1} = -\frac{L_d}{\Delta t} \{1 - S_t^{-1}\}\, i_d$$



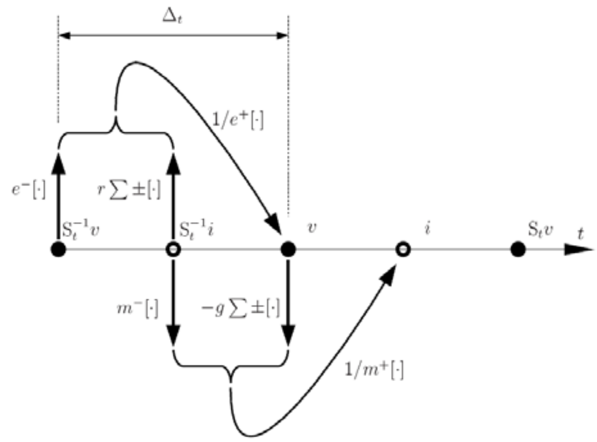
$$\{1 - S_{d+1}^{-1}\}\, S_t^{-1} i_{d+2} - \{1 - S_{d+2}^{-1}\}\, S_t^{-1} i_{d+1} + S_t^{-1} i_{\mathrm{src},d} = + \frac{C_d}{\Delta t} \{1 - S_t^{-1}\}\, v_d$$

- Time domain tracking of EM field
- Fourier transformation after steady state

Time Iteration

$$v_d = \frac{e_d^- \mathrm{S}_t^{-1} v_d + r_d \left(\left\{ 1 - \mathrm{S}_{d+1}^{-1} \right\} \mathrm{S}_t^{-1} i_{d+2} - \left\{ 1 - \mathrm{S}_{d+2}^{-1} \right\} \mathrm{S}_t^{-1} i_{d+1} + \mathrm{S}_t^{-1} i_{\mathrm{src},d} \right)}{e_d^+},$$

$$i_d = \frac{m_d^- \mathrm{S}_t^{-1} i_d - g_d \left(\left\{ \mathrm{S}_{d+1} - 1 \right\} v_{d+2} - \left\{ \mathrm{S}_{d+2} - 1 \right\} v_{d+1} \right)}{m_d^+},$$



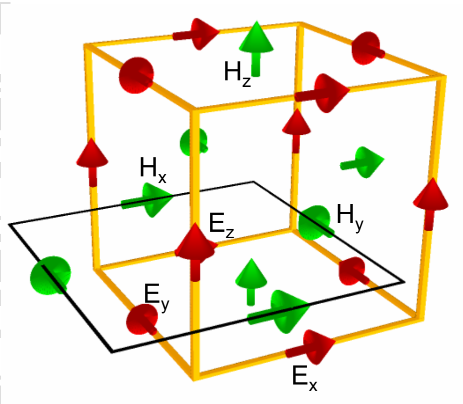
Stability (1)

Time Step limited by **spatial resolution**

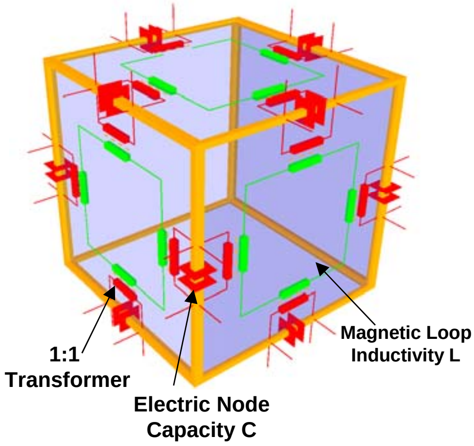
$$\Delta_t \le \min\left[\frac{\sqrt{\varepsilon_r}}{c_0} \cdot \frac{1}{\sqrt{\left(\frac{1}{\Delta x}\right)^2 + \left(\frac{1}{\Delta y}\right)^2 + \left(\frac{1}{\Delta z}\right)^2}} \right]$$

- Classical FDTD stability criterion
- Small details: long simulation time

FDTD Equivalent circuit



Equivalent Circuit



Stability (2)

Est. biggest Eigen-Frequency of Equivalent Circuit →

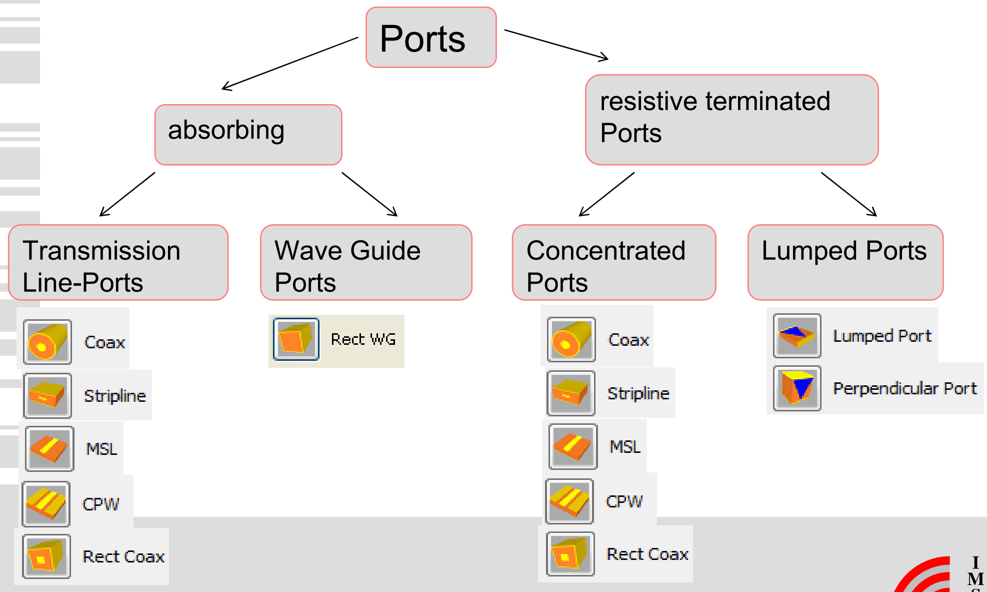
$$\omega_{\max} = \max \sqrt{\frac{1}{C}\sum_L \frac{4}{L}}$$

Limitation of time stepping scheme →

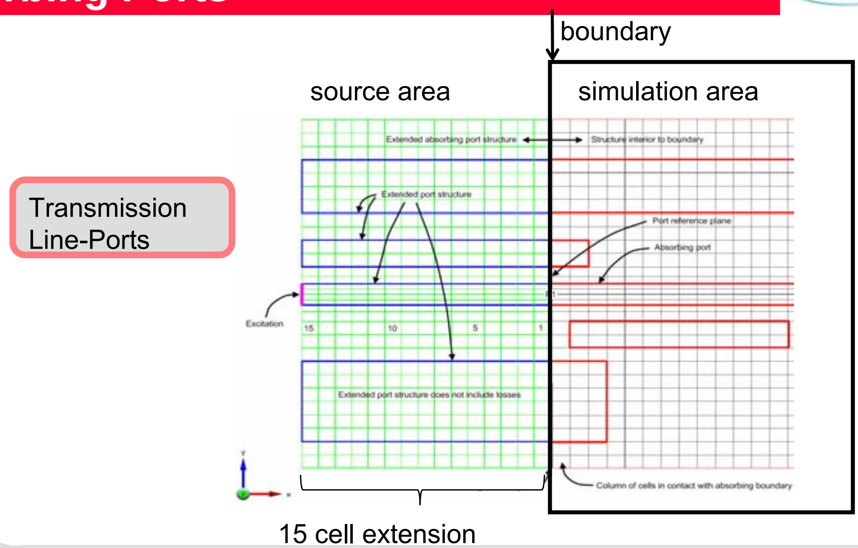
$$\Delta_t \leq \frac{2}{\omega_{\max}}$$

- Improvement Factor 2...20 in strongly graded meshes (e.g. to resolve thin metallisations or nitride sheets)

Ports in FDTD

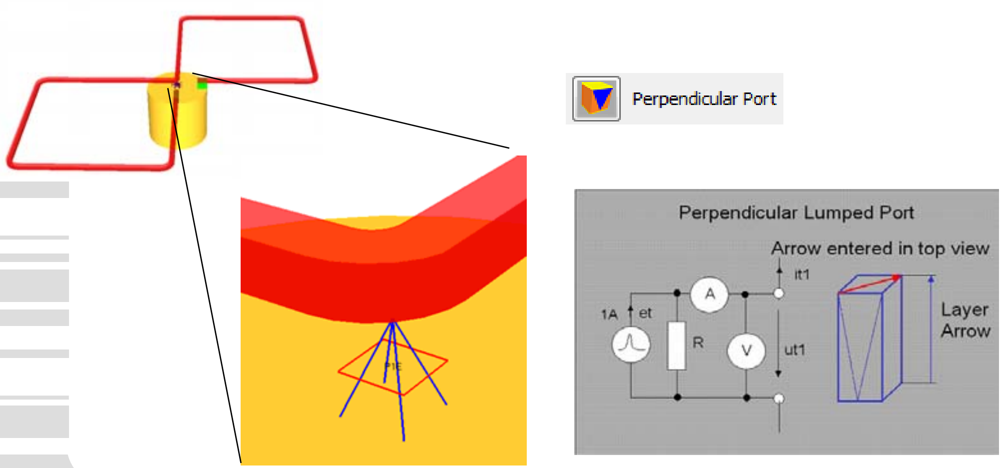


Absorbing Ports



Ports must be placed at boundary

Resistive terminated ports



Ports can be placed inside simulation area

Boundaries

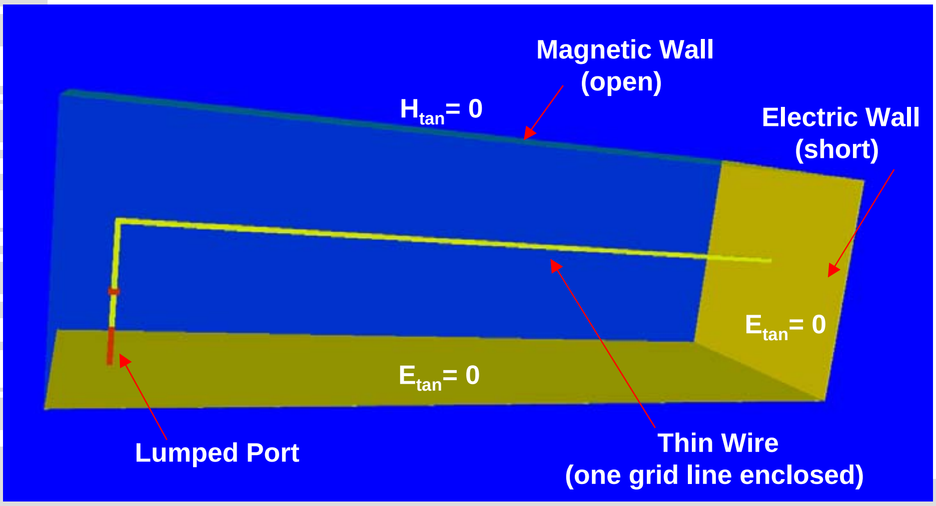
Boundary Condition

- absorbing
  - PML
  - sheet
- hard
  - electric
  - magnetic

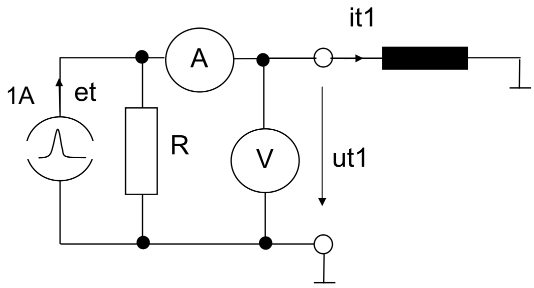
PML (Perfectly matched layer): consists of several lossy layers which are matched to each other

sheet : resistive sheet with n x 377 Ω R_{square}, faster than PML, only for perpendicular waves no reflection

Wire example (1)



Wire example (2)

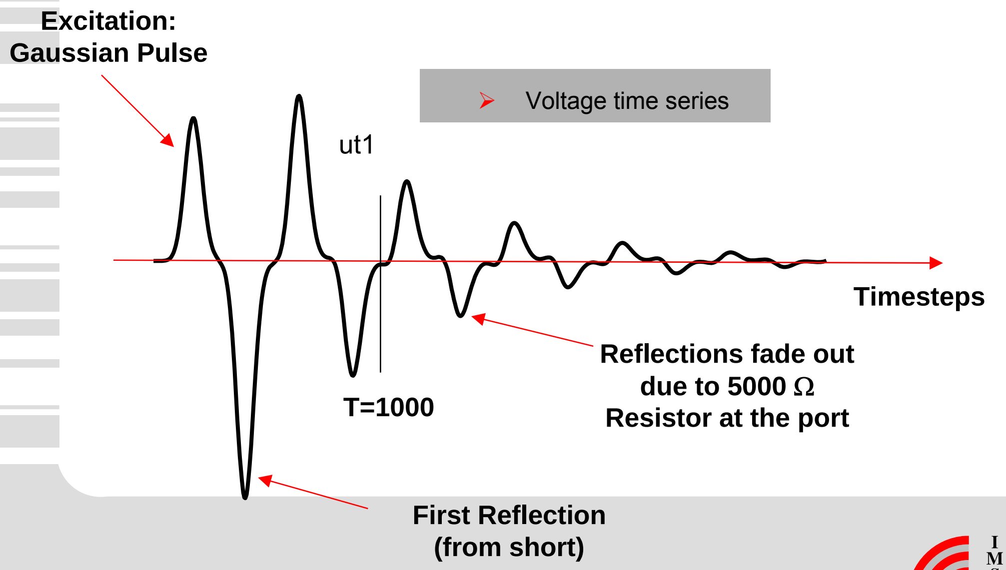


$$et = \exp\left[-\left(\frac{t-t_0}{\tau}\right)^2\right]$$

- Equivalent circuit for port
- Heavyside transformation for separating incident and reflected wave

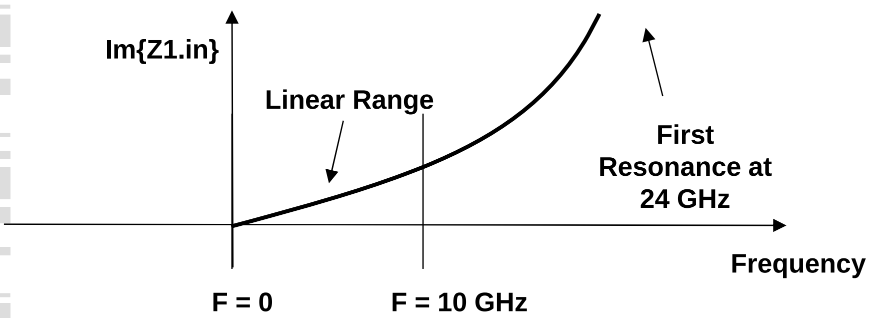
Wire example (3)

- Voltage time series



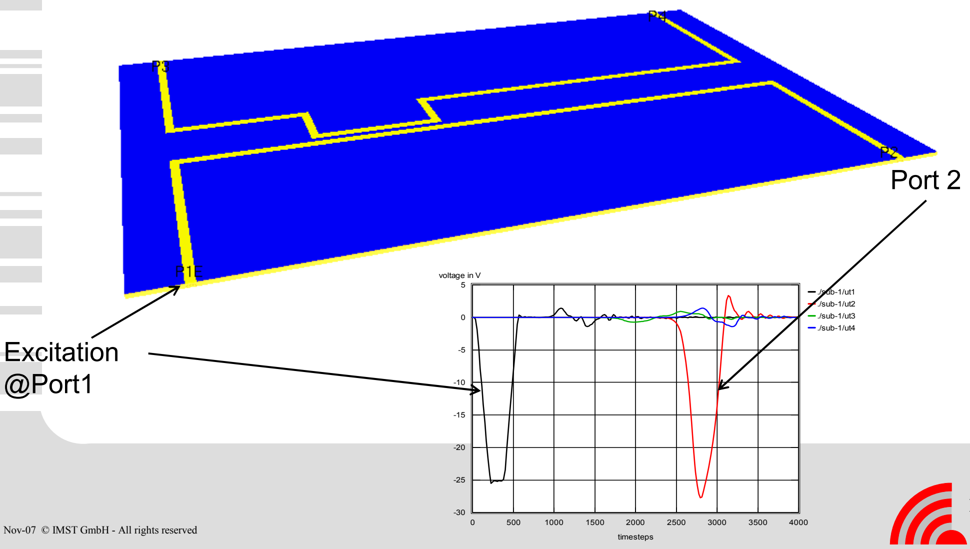
Wire example (4)

- Discrete Fourier Transformation
- Frequency dependent results: impedances, S-parameters, ...



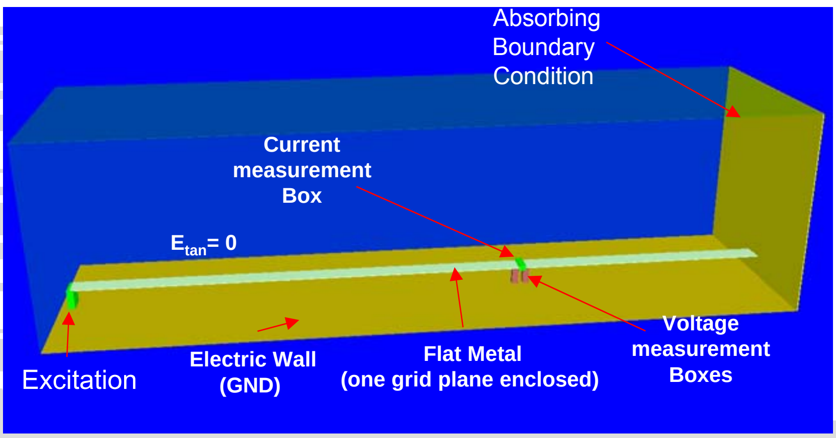
- Impedance at port
- Inductance L = Im{Z1.in}/(jω) for low frequencies (linear range)

Time Domain Simulation: Digital Pulse



FDTD accuracy (1)

- Infinite Microstrip Transmission Line



FDTD accuracy (2)

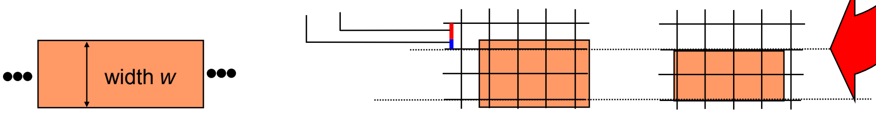
- FDTD accuracy is 2nd order, Error ~ $(\Delta_x)^2$
- Flat metallisation makes TEM transmission line parameters only 1st order accurate, Error ~ Δ_x

Cell size

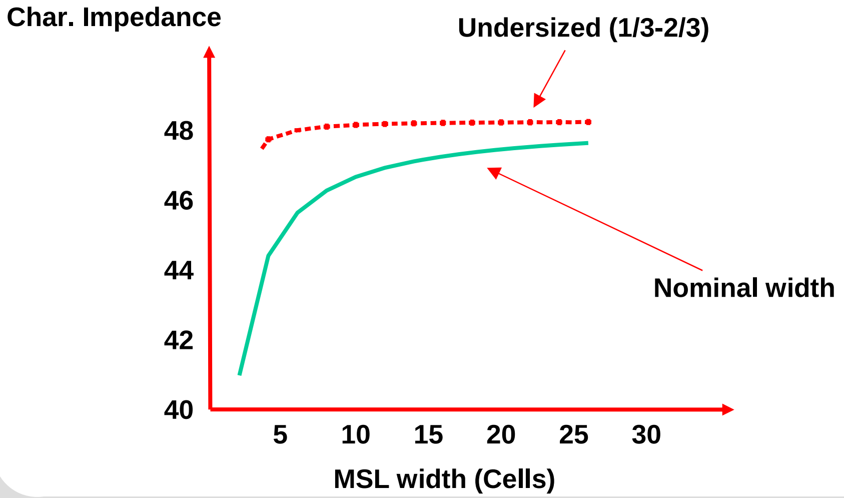
- 1/3 Δ_x Undersizing is another 1st order Error suitable for compensation.

Example:

flat metal line to be sim. **1/3-2/3 Rule mesh generation:** **width after mapping on the grid:**



FDTD accuracy (3)



FDTD loss calculation (1)

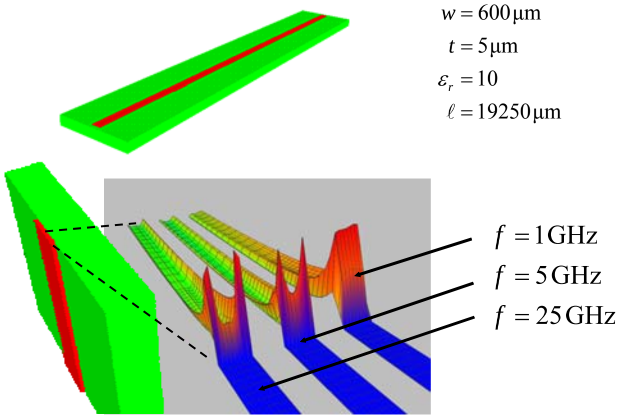
➤ Finite Microstrip Transmission Line

Skin effect:

$$R_{square} = \frac{1}{2\sigma a}$$

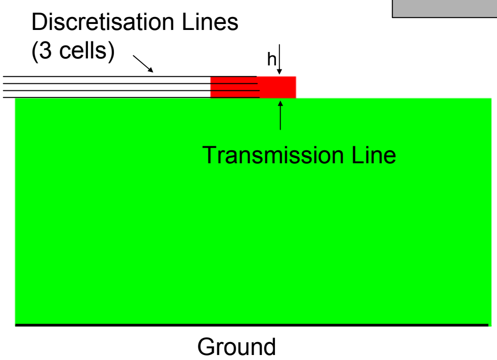
$$a(f) = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$h = 635\,\mu m$
$w = 600\,\mu m$
$t = 5\,\mu m$
$\varepsilon_r = 10$
$\ell = 19250\,\mu m$



FDTD loss calculation (2)

Modelling of the skin effect:



3 options to take into account the skin effect:

1. Resolve Material
   - Fine discretisation. ☹
   - Long simu time ☹
2. Sheet: Narrow band model
   - Resolve with 0 cells (flat) ☺
   - Narrow band ☹
3. Sheet: Broad band model
   - Resolve with 0 or 1 cells (flat) ☺
   - Double sided skin effect ☺
   - Broad band modell ☺

FDTD loss calculation (3)

$$R'\ell = \frac{\ell}{\sigma w t} = 0.115\Omega$$

$$s21(f \rightarrow 0) = -0.02\,\text{dB}$$

Resolved Material:

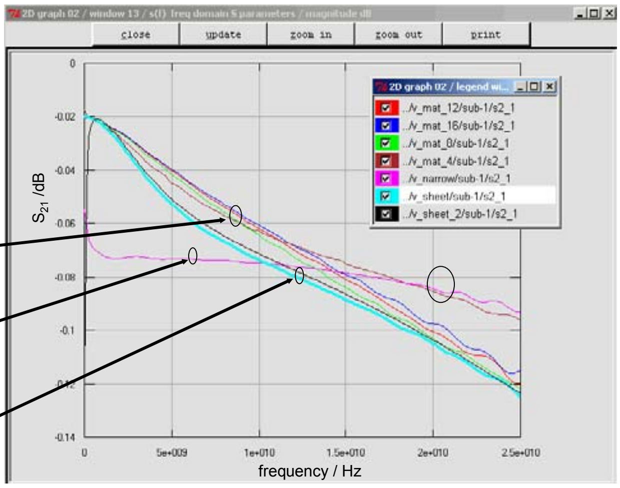
4 cells	200.000 steps
8 cells	400.000 steps
12 cells	600.000 steps
16 cells	800.000 steps

Sheet: Narrow band model:

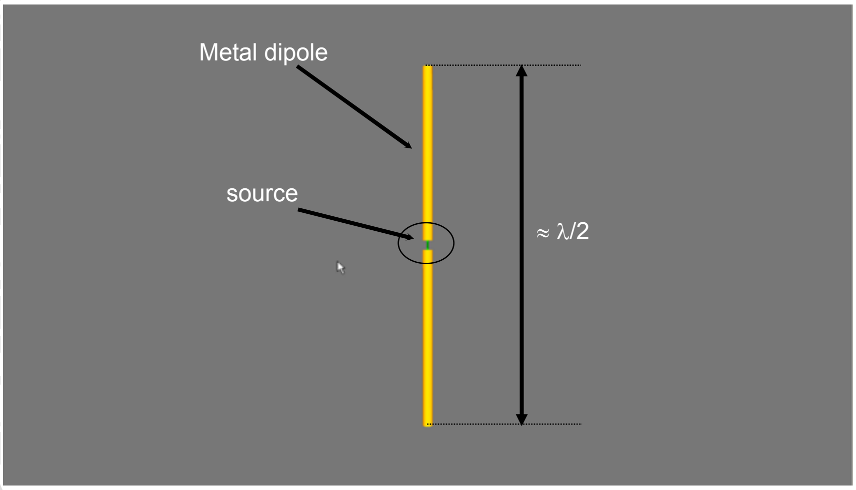
0 cell	3.700 steps

Sheet: Broad band model:

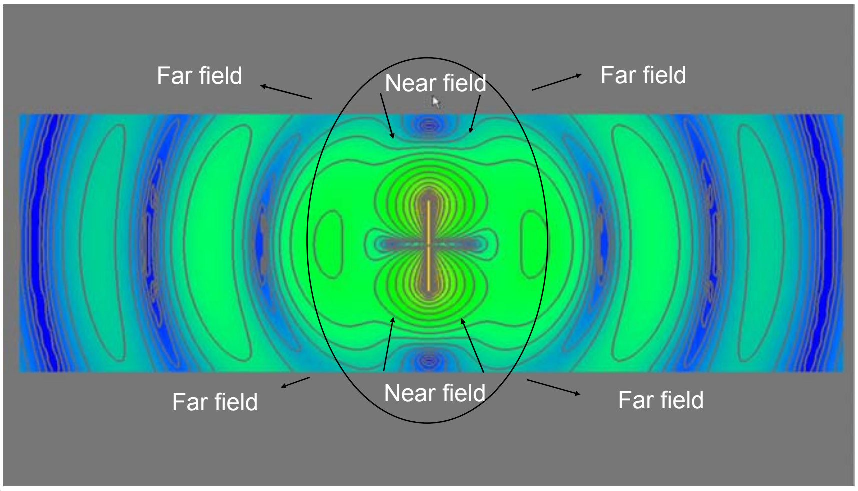
0 cell	3.700 steps (cyan)
1 cell	12.000 steps (black)



Near to far field transformation

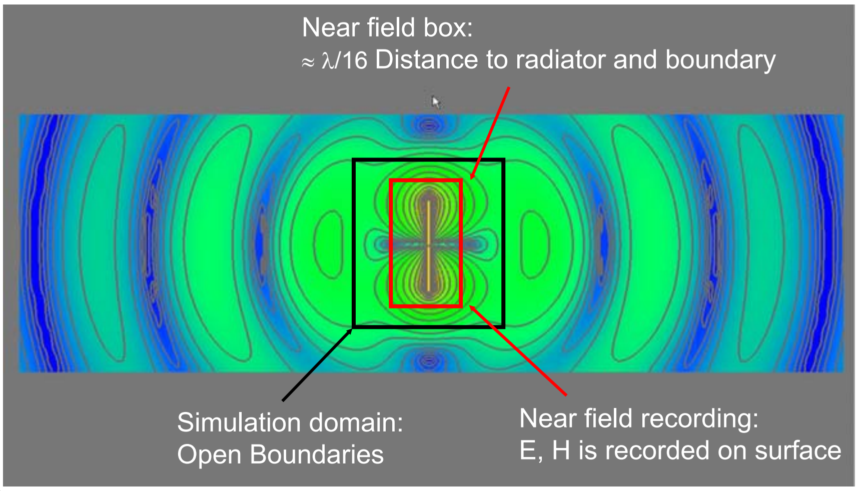


Near to far field transformation



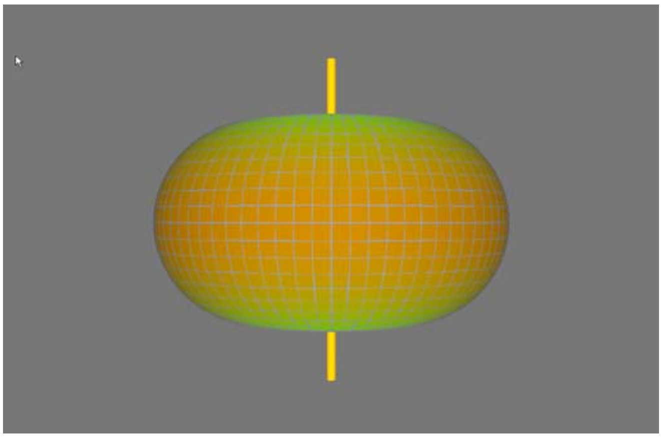
Near field: local resonance (reactive)
Far field: Waves are relieving (radiation)

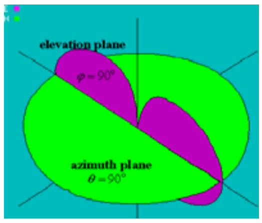
Near to far field transformation



Near field: local resonance (reactive)
Far field: Waves are relieving (radiation)

Near to far field transformation





Non-uniform radiation: Certain directions are preferred (Directivity=D(θ,φ)

Speed Optimization

- ✓ **Usage of the new processors (Pentium IV, Athlon 64, Athlon XP, Xeon) 3D calculation extensions (SIMD)**
  - multiple floating point operations each processor cycle
  - usage of multi-level processor cache
- ✓ **Optimized C-code generated for each simulation / structure**
  - only the necessary equations are solved in the specific simulation region
- ✓ **Efficient parallel computing on Multicore CPU's**
  - Innovative usage of multiple core / CPU cache for parallel FDTD calculations
  - simulation time reduced by the factor 10 … 20

⇒ **Today's performance:** ~900 Mcells/s @ Xeon-Architecture

Optimized assembler-code for Pentium 4

Precalculated RAM distance

```
movaps  239904(field),xmm0
movaps  240176(field),xmm6
subps                xmm6,xmm0
movaps  119952(field),xmm1
movaps  125664(field),xmm7
subps                xmm1,xmm7
addps                xmm0,xmm7
```

$$Ex'^{n}_{k} = Ex'^{n-1}_{k} + c_k d_k \left(\Delta Hz'^{n-\frac{1}{2}}_{k} + \Delta Hy'^{n-\frac{1}{2}}_{k} \right)$$

$$k = \ell, \ell+1, \ell+2, \ell+3$$

Sum up 4x4 H-components

```
movaps     48(coeff ),xmm3
movaps     48(denorm),xmm4
mulps                xmm4,xmm3
```

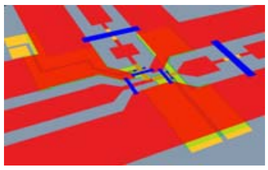
Denormalize Node Capacities (less RAM access)

```
mulps                xmm3,xmm7
movaps  359856(field),xmm2
addps                xmm7,xmm2
movaps               xmm2,359856(field)
```

Update 4 E-components

Simulation Speed on multicore CPU's

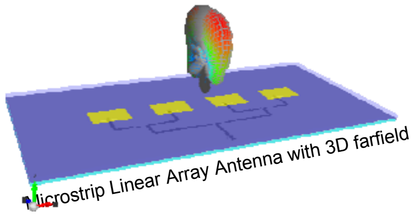
Computer	Performance
2 x Xeon 5350 2.66 GHz	900e6 cells/s
2 x Xeon 5150 2.66 GHz	500e6 cells/s
1 x Intel Core 2 duo E4500	300e6 cells/s
1 x Xeon 5150 2.66 GHz	245e6 cells/s
AMD 64 X2 4200+ 2.2 GHz	136e6 cells/s
Pentium D 2.8 GHz	190e6 cells/s
P4, 3.4 GHz	102e6 cells/s

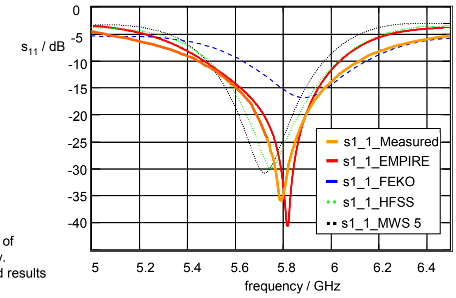


MEMS SPDT switch

- **modern 64 bit PC's allow problem sizes up to 48 GB**
- **ultra fast parallel FDTD simulation on multicore & multi CPU PC's reduces simulation time strongly**

EU-Network of Excellence: ACE – Benchmark





Results from ACE benchmark comparison:

The ACE – Network of Excellence is funded within the 6. Frame Programme of The European Union. ACE concentrates on Antenna Theory and Technology. Results of the work are published and disseminated. Please refer for detailed results at http://www.antennasvce.org -> Softlab -> Run1)

Software	EMIPRE v. 4.2	CST MWS 5	Ansoft HFSS v. 9	FEKO	MR/FDTD IETR Rennes	IMELSI IETR	FP-TLM LEAT	IE3D
Total CPU time	**10 min**	**105 min**	**1879 min**	**91 min**	**222 min**	**780 min**	**75min**	**8 min** *
Simulation Setup	3D	3D	3D	3D	3D	3D	3D	2.5D *
Type of machine	Desktop PC	Desktop PC	Desktop PC	Desktop PC	Desktop PC	Desktop PC	Parallel CPU WS	Desktop PC
CPU	PIV 3.4 GHz	PIV 1.7 GHz	PIV 2.4 GHz	PIV 3 GHz	AMD Athlon 3500+	PIV 3 GHz	16 x 1.3 GHz	PIV 3 GHz
Method	FDTD	FITD	FEM	MOM	FDTD	FDTD	TLM	MOM
Used RAM	180 MB		512 MB	143 MB	370 MB	1.3 GB	16 x 466 MB	29 MB

* use of infinite dielectric substrate & infinite metal planes to reduce simulation time

Conclusions

- Maxwell‘s Equations discretized in space and time
- EMPIRE yields an improved Stability Criterion for strongly graded meshes
- Fast & Accurate flat metal simulation with 1/3 cell undersizing
- EMPIRE is optimized to efficiently use modern Computers‘ resources