Efficient Analysis of Complex Modes in Cylindrical Photonic Crystal

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Cylindrical photonic crystal is a topic of active research because of its potential importance in fiber-optic communications [1], nonlinear devices [2]. The modal properties of cylindrical photonic crystal have been extensively investigated using the finite element method combined with PML [3], the finite difference frequency domain method, the multipole method [4]. These numerical methods could be versatilely applied to various microstructured configurations but they are computationally intensive.

In this paper, we shall present a novel full-wave rigorous approach for the vector fields in cylindrical photonic crystals, which consists of layered cylindrical arrays of circular rods symmetrically distributed on each of concentric (eccentric) circular cylindrical surfaces. The method is computationally fast and easy to implement for a wide class of cylindrical photonic crystals. The proposed approach introduces a cylindrical layer model to the array, extracts the reflection and transmission matrices of a cylindrical periodic layer, and then obtains the characteristics of the whole layer structure by using a recursive algorithm [5, 6]. In our formalism we take into account all cylindrical Floquet modes and their interactions through the scattering by each cylindrical layer. In the case of the modal analysis of the guided waves without any initial excitation, we could assume a unique symmetric property of the mode field distribution inherent to the periodicity of the circular rods. In the work we calculate the complex propagation constant for hexagonal arrays and the results will be presented at the symposium.

References


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Introduction

• Motivation.

• Novelty of the Work.

• Formulation of the Problem.

• Numerical Results and Discussions.
Motivation

- All-optical logic gates, which are responsible for various logical operations in all-optical circuits, play a key role in ultrafast optical signal processing.

- We demonstrated the realization of true all-optical NOT, AND and NAND logic gates using gap-solitons in photonic crystal waveguides composed of an experimentally feasible planar air-hole type hexagonal structure.

\[ n_0 = 2.95 \text{ (linear refractive index)} \]
\[ r = 0.32h \text{ (radius of the rods)} \]
\[ w = 1.73h \text{ (width of the waveguide)} \]

Bandgap for H-modes \((H_z, E_x, E_y)\). Air-Hole type PhCs. Length of the device is 30h.


Motivation

A key element in the working concept of the proposed all-optical logic gates is the virtually “perfect digitalization” of the involved time-domain signals. All investigated gate topologies operate with temporal bandgap solitons having stable pulse envelopes during signal processing, which is one of main advantages of the proposed working concept of the device.

Realization of fully optical NOT logic gate

Similar characteristics between I/O has been observed (key element for true all-optical logic gates). Relative error between the output and input signals (its amplitude and width) is about 7% - 9%.
Motivation

In the proposed setup, there is no need to amplify the output signal after each logic operation, and can be directly use it as a new input signal for another logical operation.

Can we realize logical operations in Photonic Crystals with cylindrical symmetry?

Novelty of the Work

**Cylindrical Photonic Crystals**, which are formed by circular rods periodically distributed on layered concentric or eccentric circular [1].

1. For isotropic scatterer the reflection matrix of single planar layer always satisfies the following equality: $R_{v,v-1} = R_{v-1,v}$ (Reciprocity Relation).

2. Each Floquet mode is excited only in some particular frequency range.

1. Since the reflection matrices are expressed in terms of the cylindrical waves, even for isotropic scatterer: $R_{v,v-1} \neq R_{v-1,v}$.

2. All orders of cylindrical harmonic waves are excited. However, due to the resonances and stopbands nature some of them are enhanced, whereas others are strongly suppressed.

We have developed a semi-analytical method, which can be applied to the **guiding, scattering and radiation problems** in cylindrical periodic (or EBG) structures.

The approach uses:

- Transition Matrix (T-matrix) of a circular rod;
- Translation matrices for cylindrical waves;
- Reflection and transmission matrices based on cylindrical waves for each layer;
- Generalized reflection and transmission matrices for multi-layered structure.

For the incidence of outgoing wave with $\bar{c}^{(v-1)}$

$$\bar{b}^{(v-1)} = \begin{bmatrix} b^{e(v-1)} \\ b^{h(v-1)} \end{bmatrix} = R_{v-1,v} \cdot \begin{bmatrix} c^{e(v-1)} \\ c^{h(v-1)} \end{bmatrix} = R_{v-1,v} \cdot \bar{c}^{(v-1)}$$

$$\bar{c}^{(v)} = \begin{bmatrix} c^{e(v)} \\ c^{h(v)} \end{bmatrix} = F_{v,v-1} \cdot \begin{bmatrix} c^{e(v-1)} \\ c^{h(v-1)} \end{bmatrix} = F_{v,v-1} \cdot \bar{c}^{(v-1)}$$

For the incidence of incoming wave with $\bar{b}^{(v)}$

$$\bar{b}^{(v-1)} = \begin{bmatrix} b^{e(v-1)} \\ b^{h(v-1)} \end{bmatrix} = F_{v-1,v} \cdot \begin{bmatrix} b^{e(v)} \\ b^{h(v)} \end{bmatrix} = F_{v-1,v} \cdot \bar{b}^{(v)}$$

Formulation of the Problem

For dielectric circular rods:

$$R_{v-1,v} = \begin{bmatrix} R_{v-1,v}^{ee} & R_{v-1,v}^{eh} \\ R_{v-1,v}^{he} & R_{v-1,v}^{hh} \end{bmatrix}, \quad R_{v,v-1} = \begin{bmatrix} R_{v,v-1}^{ee} & R_{v,v-1}^{eh} \\ R_{v,v-1}^{he} & R_{v,v-1}^{hh} \end{bmatrix}$$

$$F_{v,v-1} = \begin{bmatrix} F_{v,v-1}^{ee} & F_{v,v-1}^{eh} \\ F_{v,v-1}^{he} & F_{v,v-1}^{hh} \end{bmatrix}, \quad F_{v-1,v} = \begin{bmatrix} F_{v-1,v}^{ee} & F_{v-1,v}^{eh} \\ F_{v-1,v}^{he} & F_{v-1,v}^{hh} \end{bmatrix}$$

For perfect conductor circular rods:

$$R_{v-1,v} = \begin{bmatrix} R_{v-1,v}^{ee} & 0 \\ 0 & R_{v-1,v}^{hh} \end{bmatrix}, \quad R_{v,v-1} = \begin{bmatrix} R_{v,v-1}^{ee} & 0 \\ 0 & R_{v,v-1}^{hh} \end{bmatrix}$$

$$F_{v,v-1} = \begin{bmatrix} F_{v,v-1}^{ee} & 0 \\ 0 & F_{v,v-1}^{hh} \end{bmatrix}, \quad F_{v-1,v} = \begin{bmatrix} F_{v-1,v}^{ee} & 0 \\ 0 & F_{v-1,v}^{hh} \end{bmatrix}$$

Generalized Reflection and Transmission Matrices

**For antenna problem**

\[ \overline{R}_{v-1,v} = R_{v-1,v} + F_{v-1,v} \overline{R}_{v,v+1} \overline{\Gamma}_{v,v-1} \]
\[ \overline{\Gamma}_{v,v-1} = (I - R_{v,v-1} \overline{R}_{v,v-1})^{-1} F_{v,v-1} \]
\[ \overline{R}_{N,N+1} = 0 \]

\[ \overline{R}_{v-1,v} : \text{ Generalized reflection matrix viewed from region } (v - 1) \text{ to the whole outer regions from (v) to (N)} \]

\[ \overline{F}^{(N)} = \overline{\Gamma}_{N,N-1} \overline{\Gamma}_{N-1,N-2} \ldots \overline{\Gamma}_{2,1} \overline{\Gamma}_{1,0} \]

**For guiding problem**

\[ \overline{R}_{v,v-1} = R_{v,v-1} + F_{v,v-1} \overline{\Gamma}_{v-1,v-2} \overline{R}_{v-1,v-2} F_{v-1,v} \]
\[ \overline{\Gamma}_{v-1,v-2} = (I - \overline{R}_{v-1,v-2} \overline{R}_{v-1,v})^{-1} \]
\[ \overline{R}_{2,1} = R_{2,1} \]

\[ \overline{R}_{v-1,v-2} : \text{ Generalized reflection matrix viewed from region } (v - 1) \text{ to the whole inner regions from (v - 1) to (0)} \]
Formulation of the Problem

Schematics for guidance, scattering and radiation problems.

Guiding problems

Scattering problems

Radiation problems

Numerical Results and Discussions

Microstructure Optical Fiber

Background medium is pure Silica. Air-holes are drilled in Silica.

Effective refractive index of cladding is lower than that of core. **TIR effect.**

No Bandgap Region was formed.

![Graph 1](image1)

![Graph 2](image2)
Numerical Results and Discussions

Effective refractive index of cladding is higher than that of core. **No TIR effect.**

- \( n_{\text{hole}} = 1.485 \)
- \( n_{\text{cladding}} = 1.449 \)
- \( d_{\text{hole}} = 2.3 \) (micron)
- \( r_{\text{hole}} = 5.7 \) (micron)

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